RECTANGULAR OFFSET STRIP-FIN HEAT EXCHANGER LUMPED PARAMETERS DYNAMIC MODEL

Miguel Henrique Teruel, miguel.teruel@embraer.com.br
EMBRAER, Av. Brigadeiro Faria Lima, 2170, São José dos Campos, São Paulo, Brasil
ITA, Praça Marechal Eduardo Gomes, 50, São José dos Campos, São Paulo, Brasil

Celso Yukio Nakashima, celso.nakashima@embraer.com.br
EMBRAER, Av. Brigadeiro Faria Lima, 2170, São José dos Campos, São Paulo, Brasil

Pedro Paglione, paglione@ita.br
ITA, Praça Marechal Eduardo Gomes, 50, São José dos Campos, São Paulo, Brasil

Abstract. *This paper presents a lumped parameters model for predicting the temperature dynamics of heat exchanger core, as the out temperature dynamics of both the hot and cold flows. Fin and core's geometry are used in empirical correlations for providing mass flow and thermal resistances to the model.*

Keywords*: strip-fin, heat exchanger, dynamic mode, lumped*

1. INTRODUCTION

There are several types of compact heat exchanger, but the offset strip-fin has been the most widely used fin geometry for industries that require lightweight high-performance exchangers, due to its high heat transfer relative to heat exchanger volume, an important characteristic when considering the often reduced space available for its positioning.

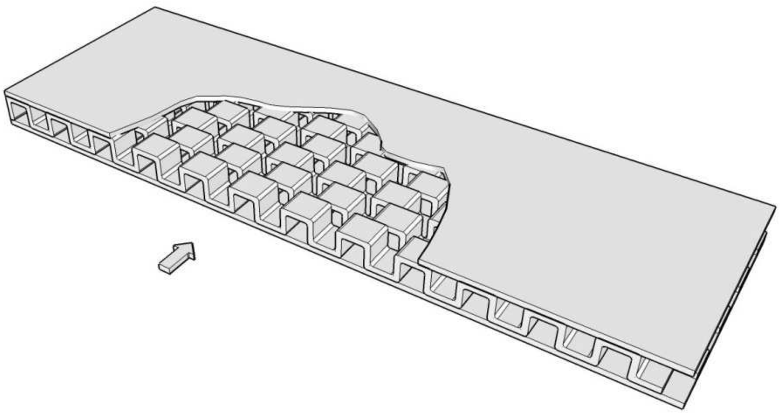

Figure 1. One layer of an offset strip-fin

Due to the complex nature of the flow in this type of heat exchanger, empirical correlations have been developed for over 60 years now, with the first friction factor and Colburn modulus data being presented by (Norris and Spofford, 1942) for 3 offset surfaces. Since this date, this correlations have been constantly updated, and as reported by (Shah and Sekulic, 2003), the most comprehensive correlations available at this date were provided by (Manglik and Bergles, 1990). Based on the correlation for the friction factor, the mass flow through each line can be obtained, and using the correlation for the Colburn modulus and the mass flow, a model is proposed for representing all thermal resistances, and an equivalent thermal resistance from the fluid to the core is obtained. The development for the mass flow calculation is presented in Section 2, and for the thermal resistance in Section 3.

The dynamic model is then presented in section 4, where previously obtained results will be joining additional developments for calculating the heat exchanger's core temperature, and finally, the outflow temperature of the both the cold and hot lines.

2. EMPIRICAL CORRELATIONS

The once "state-of-the-art" (Shah and Webb, 1983) empirical correlations for predicting Colburn modulus and friction factors provided by (Wieting, 1975), as many other correlations, were replaced by those provided (Manglik and Bergles, 1990). As the authors state: "the few empirical correlations that are available inadequately describe the trends in the data and lack a logical theoretical basis" (Manglik and Bergles, 1990). These correlations predict the experimental data of 18 test cores within ± 20% for $120 \leq Re \leq 10^4$, and, even though obtained for air, since the *j* factor takes into account variations in the Prandtl number (Pr), they should be valid for 0,5 < Pr < 15 (Shah and Sekulic, 2003).

Figure 2 presents a frontal view of the offset strip-fin view, and the fin parameters, which will be referenced throughout this paper:

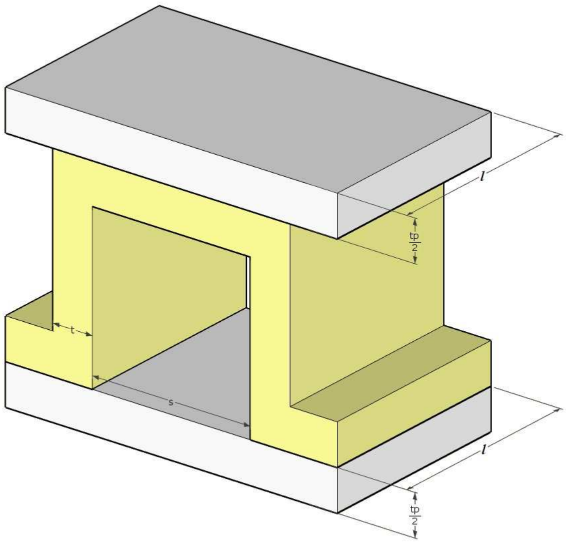

Figure 2. Geometric parameters of the offset strip-fin

- ***s***: Transverse spacing (free flow width).
- ***h***: Free flow height.
- ***t***: Fin thickness.
- ***l:*** Fin length.

(Manglik and Bergles, 1990) also provide a new definition of hydraulic diameter (D_h), which considers the mean flow velocity – instead of maximum velocity as in (Joshi and Webb, 1987) – and also accounts for both the vertical and lateral fin edges, extending the understanding of (London and Shah, 1968) and (Joshi and Webb, 1987), who considers only vertical edges. Eq. (1) provides this hydraulic diameter definition.

$$D_h = \frac{4shl}{2(sl+hl+th)+ts} \quad (1)$$

2.1. Mass Flow

The correlation that (Manglik and Bergles, 1990) have obtained for the friction factor is in the form of a power-law of the type presented in Eq. (2).

$$f = K_1(Re)^{a1}(\alpha)^{a2}(\delta)^{a3}(\gamma)^{a4} \quad (2)$$

- ***K_1, a1, a2, a3, a4***: Power-law coefficients.
- ***Re***: Reynolds number.
- $\boldsymbol{\alpha = s/h}$: Aspect ratio.
- $\boldsymbol{\delta = t/l}$**:** Ratio.
- $\boldsymbol{\gamma = t/s}$**:** Ratio.

For $Re \leq Re^*$, laminar flow region:

$$f = 9{,}6243(Re)^{-0{,}7422}(\alpha)^{-0{,}1856}(\delta)^{0{,}3053}(\gamma)^{-0{,}2659} \quad (3)$$

For $Re \geq Re^* + 1000$, turbulent flow region:

$$f = 1{,}8699(Re)^{-0{,}2993}(\alpha)^{-0{,}0936}(\delta)^{0{,}6820}(\gamma)^{-0{,}2423} \quad (4)$$

Re^* is defined as presented in Eq. 5, from (Joshi and Webb, 1987):

$$Re^* = 257\left(\frac{l}{D_h}\right)^{1,23}\left(\frac{t}{l}\right)^{0,58} D_h\left[t + 1{,}328\left(\frac{Re}{lD_h}\right)^{-0,5}\right]^{-1} \tag{5}$$

For converting the friction factor into mass flow, we are required to use the definition of the average Fanning friction factor, as presented in Eq. (6), and to convert the mean fluid velocity into mass flow, using Eq. (7):

$$f = \frac{\Delta P}{2}\frac{D_h}{L}\frac{1}{\rho u_m^2} \tag{6}$$

- $\boldsymbol{\Delta P}$: Pressure loss in the heat exchanger.
- $\boldsymbol{L}$: Total length in the direction of the flow.
- $\boldsymbol{\rho}$: Fluid density.
- $\boldsymbol{u_m}$: Mean fluid velocity.

$$u_m = \frac{\dot{m}}{\rho A} \tag{7}$$

- $\dot{\boldsymbol{m}}$: Mass flow.
- $\boldsymbol{A}$: Total free flow area.

Replacing the mean velocity in Eq. (6) by Eq. (7) and isolating the mass flow, we have Eq. (8):

$$\dot{m} = A\sqrt{\frac{1}{f}\frac{\rho\Delta P}{2}\frac{D_h}{L}} \tag{8}$$

Since the friction factor is dependent on the Reynolds number, which in turn depends on the mass flow, which we are meant to obtain, it is required to replace the general form of the correlation of Eq. (2) in the mass flow of formulae of Eq. (8), obtaining then Eq. (9):

$$\dot{m} = A\sqrt{\frac{1}{K_1 Re^{a1}\alpha^{a2}\delta^{a3}\gamma^{a4}}\frac{\rho\Delta P}{2}\frac{D_h}{L}} = A\sqrt{\frac{1}{K_1\left(\frac{\dot{m}D_h}{\mu A}\right)^{a1}\alpha^{a2}\delta^{a3}\gamma^{a4}}\frac{\rho\Delta P}{2}\frac{D_h}{L}} \tag{9}$$

- μ: Fluid viscosity.

Isolating the mass flow again, we have Eq. (10), that provides the mass flow as a function of the remaining elements of the friction power-law, fluid density, hydraulic diameter, length in the flow direction and the pressure differential in the heat exchanger.

$$\dot{m} = A\left[\frac{1}{K_1\mu^{-a1}\alpha^{a2}\delta^{a3}\gamma^{a4}}\frac{\rho\Delta P}{2}\frac{D_h^{(1-a1)}}{L}\right]^{1/(2+a1)} \tag{10}$$

2.2. Thermal Resistance

Now that the necessary calculations for the mass flow have been provided, we are able to fully develop the equations for the thermal resistance. First, the heat transfer coefficient is determined from an empirical equation in Section 2.2.1, and then used in Section 2.2.2 for calculating the convection thermal resistance. The later section also presents conduction thermal resistance and equivalent resistance model.

2.2.1 Convection Coefficient

The Colburn modulus is defined as:

$$j = St.Pr^{2/3} \tag{11}$$

- j: Colburn modulus.
- St: Stanton number.

- Pr: Prandtl number.

Using the dimensionless numbers definitions:

$$St = \frac{Nu}{Re.Pr} \tag{12}$$

$$Nu = \frac{h.D_h}{K_f} \tag{13}$$

$$Pr = \frac{c_p\mu}{K_f} \tag{13}$$

- h_e: Convection heat transfer coefficient.
- K_f: Fluid thermal conductivity.
- c_p: Fluid specific heat at constant pressure.
- μ: Fluid viscosity.

Rearranging Eq. (11) using Eq. (12), Eq. (13) and Eq. (14), isolating the convection coefficient:

$$h_e = j.Re.\left(\frac{c_p\mu}{K_f}\right)^{1/3}\frac{K_f}{D_h} \tag{14}$$

For the convection coefficient it is not necessary to manipulate the correlation, so the Colburn modulus is simply calculated and fed to Eq. (14). Eq. (15) and Eq. (16) presents the (Manglik and Bergles, 1990)'s correlation for the laminar and turbulent regions:

For $Re \leq Re^*$, laminar flow region:

$$j = 0,6522(Re)^{-0,5403}(\alpha)^{-0,1541}(\delta)^{0,1499}(\gamma)^{-0,0678} \tag{15}$$

For $Re \geq Re^*$, turbulent flow region:

$$j = 0,2435(Re)^{-0,4063}(\alpha)^{-0,1037}(\delta)^{0,1955}(\gamma)^{-0,1733} \tag{16}$$

2.2.2 Equivalent Thermal Resistance

Heat transfer will occur in the fin through five different types of resistances, as presented in Figure 3. This resistances model both convection (using the heat transfer coefficient calculated previously) and conduction. After defining the equations for each resistance, a thermal circuit is built, and an equivalent thermal resistance is calculated.

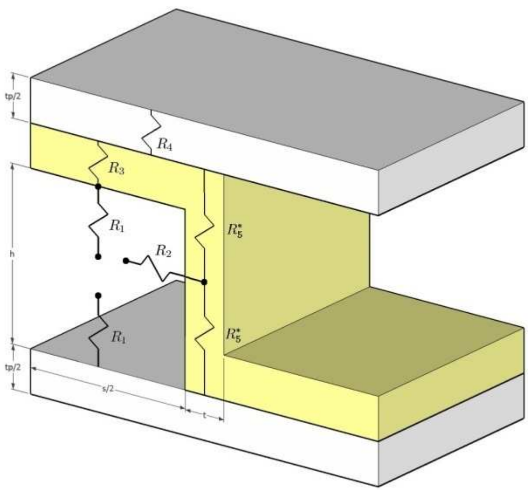

Figure 3. Fin thermal resistances

- R_1: Convection from fluid to plate/fin base.
- R_2: Convection from fluid to fin projection.
- R_3: Conduction through fin base.
- R_4: Conduction through half the plate thickness.
- R_5^*: Conduction through fin projection.

Resistances R_2 and R_5^* are presented in Figure 3 for illustration purposes only. An additional model will be used for calculating this resistances. The other three resistances can be obtained directly from the model presented in Figure 2. These are:

$$R_1 = \frac{1}{h_e N_f 2 \frac{s}{2} l} = \frac{1}{h_e N_f s l} \quad (17)$$

$$R_3 = \frac{t}{K_n N_f 2 \frac{s}{2} L_a} = \frac{t}{K_n N_f s l} \quad (18)$$

$$R_4 = \frac{t_p/_2}{K_p N_f 2 \frac{s}{2} L_a} = \frac{t_p}{2. K_p N_f s L_a} \quad (19)$$

- N_f: Total number of fins in the line (cold or hot).
- K_n: Fin thermal conductivity.
- K_p: Plate thermal conductivity.

The vertical edge can be rearranged for applying a model proposed in (Incropera, 2006). Therefore, the real z-shaped fin will be replaced by a T-shaped fin, with a behavior very well known from literature. Figure 4 presents this model and its parameters.

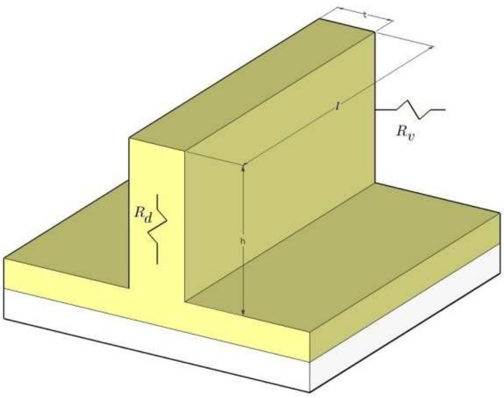

Figure 4. Fin thermal resistances

- R_v: Convection resistance.
- R_d: Conduction resistance.

In this model, the total equivalent fin resistance will be a combination of the convection and conduction resistances. Since the top of the fin, in this model, is adiabatic, it is possible to partition this fin into two equal parts, with a corrected length (L_c) half of the total fin height. The decision to partition this fin is will require that later, both fins are combined through a parallel representation.

For this model, the following equations are required:

$$L_c = \frac{h}{2} \quad (20)$$

$$A_f = 2. l. L_c \quad (21)$$

$$m = \sqrt{\frac{h.2.(l + t_a)}{K.l.t_a}} \tag{22}$$

$$\varepsilon_f = \frac{tanh(mL_c)}{mL_c} \tag{23}$$

- L_c: Corrected length
- A_f: Heat exchanging surfaces
- m: Efficiency parameter
- ε_f: Fin efficiency

Using this parameters, it is possible to define the half-fin thermal resistance, R_a, and a total fin thermal resistance, R_5, a parallel combination of the fin thermal resistance, R_a:

$$R_a = \frac{1}{h.N_f A_f \varepsilon_f} \tag{24}$$

$$R_5 = R_a//R_a = \frac{R_a.R_a}{R_a + R_a} = \frac{R_a}{2} = \frac{1}{2}\frac{1}{h.N_f A_f \varepsilon_f} \tag{25}$$

With all the resistances defined, we are ready for creating an equivalent thermal circuit for the heat transfer through the fin.

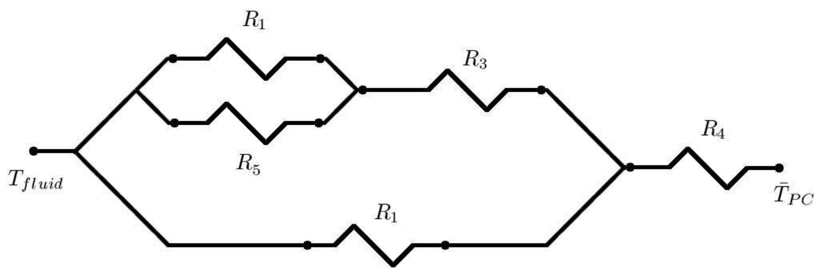

Figure 4. Equivalent thermal circuit

The equivalent thermal resistance from the fluid temperature to the mean pre-cooler mass temperature is:

$$R_{eq} = \frac{\left(\frac{R_1 R_5}{R_1 + R_5} + R_3\right) R_1}{\frac{R_1 R_5}{R_1 + R_5} + R_3 + R_1} + R_4 \tag{26}$$

This equivalent resistance should be calculated for the cold and the hot side, as they model the heat transfer from each of this fluids to the heat exchanger mass.

3. DYNAMIC MODEL

Until this point, the basis of the model have been developed, the mass flow can be calculated based on the heat exchanger geometry, as are the thermal resistances. The last part is the model itself is models the heat exchange between the cold and hot fluids to/from the core mass, and the dynamics of the temperature in this core.

Figure 5 presents the representation of this model with three main parts: hot fluid control volume, core mass and cold fluid control volume.

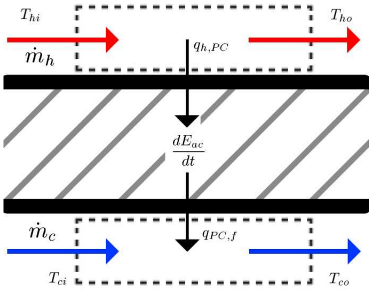

Figure 5. Core Heat Exchange Model

- $\dot{m}_h$: Hot mass flow.
- $\dot{m}_c$: Cold mass flow.
- T_{hi}: Hot inlet temperature.
- T_{ho}: Hot outlet temperature.
- T_{ci}: Cold inlet temperature.
- T_{co}: Cold outlet temperature.
- $q_{h,CM}$: Heat transferred from the hot fluid to the core mass.
- $q_{CM,c}$: Heat transferred from the core mass to the cold fluid.
- $\frac{dE_{ac}}{dt}$: Energy accumulated in the core mass.

This model now needs to be represented in a fashion suitable for use in a numerical simulation. For avoiding unnecessary repetition, the following developments will be presented for the hot line, but analog results can be obtained for the cold line (which will be presented at the end).

Representation of the heat is lost by the hot fluid due to the contact with the surface of the core mass is presented at Eq. (27). It should be noted here that all developments are based on mean temperature, as defined in Eq. (28):

$$\dot{m}_h C_p (T_{hi} - T_{ho}) = \frac{1}{R_h}(\bar{T}_h - \bar{T}_{CM}) \tag{27}$$

$$\bar{T}_h = \frac{T_{hi} + T_{ho}}{2} \tag{28}$$

The use of mean temperatures is of great importance in this model, since it allows the heat transfer, and therefore the core mass temperature, to be obtained only as a function of the inlet temperatures of the fluids.

It is required an expression for the mean hot temperature, which is not dependant on the fluid outlet temperature, which is to be determined. Isolating the hot outlet temperature in Eq. (28) and replacing into Eq. (27) yields Eq. (29), which can then be rearranged for calculating the mean hot temperature, in Eq. (30), suitable for use in the core mass dynamics equation.

$$\dot{m}_h C_p [T_{hi} - (2.\bar{T}_h - T_{hi})] = \frac{1}{R_h}(\bar{T}_h - \bar{T}_{CM}) \tag{29}$$

$$\bar{T}_h = \bar{T}_{CM} + \frac{2.R_h.\dot{m}_h.c_p(T_{hi} - \bar{T}_{CM})}{1 + 2.R_h.\dot{m}_h.c_p} \tag{30}$$

The same development can be performed for the cold line, yielding Eq. (30):

$$\bar{T}_c = \bar{T}_{CM} + \frac{2.R_c.\dot{m}_c.c_p(T_{ci} - \bar{T}_{CM})}{1 + 2.R_c.\dot{m}_c.c_p} \tag{31}$$

Eq. (30) and (31) present the mean cold and hot fluid temperatures as a function of the mean core mass temperature and known parameters, such as mass flow and equivalent thermal resistances (R_h and R_c).

Proceeding to the core mass energy dynamics, it is defined as:

$$\frac{dE_{ac}}{dt} = q_{h,CM} - q_{CM,c} = \frac{1}{R_h}(\bar{T}_h - \bar{T}_{PC}) - \frac{1}{R_c}(\bar{T}_{PC} - \bar{T}_c) = \frac{1}{R_h}(\bar{T}_h - \bar{T}_{PC}) + \frac{1}{R_c}(\bar{T}_c - \bar{T}_{PC}) \tag{32}$$

It is now required to replace Eq. (30) and Eq. (31) into Eq. (32), for obtaining the derivative as a function only of fluid inlet temperature and the mean temperature of the core mass. After some manipulation, Eq. (33) presents the result:

$$\frac{dE_{ac}}{dt} = \frac{2.\dot{m}_h.c_p}{1 + 2.R_h.\dot{m}_h.c_p}(T_{hi} - \bar{T}_{CM}) + \frac{2.\dot{m}_c.c_p}{1 + 2.R_c.\dot{m}_c.c_p}(T_{ci} - \bar{T}_{CM}) \tag{33}$$

In the last step, the energy derivative should be replaced by the temperature derivative, using Eq. (34). The result is presented in Eq. (35):

$$\frac{dE_{ac}}{dt} = m_c.c_{p,CM}\frac{d\bar{T}_{CM}}{dt} \tag{34}$$

$$\frac{d\bar{T}_{CM}}{dt} = \frac{1}{m_c.c_{p,CM}}\left[\frac{2.\dot{m}_h.c_p}{1 + 2.R_h.\dot{m}_h.c_p}(T_{hi} - \bar{T}_{CM}) + \frac{2.\dot{m}_c.c_p}{1 + 2.R_c.\dot{m}_c.c_p}(T_{ci} - \bar{T}_{CM})\right] \tag{35}$$

Integrating Eq. (35) in time, the core mass temperature becomes available, and the hot outflow temperature is then obtained using Eq. (28) and (30).

5. CONCLUSION

The model presented in this paper provides a simple solution, using only one derivative, for representing the behavior of a heat exchanger of complex geometry. Even though developed for an offset strip-fin geometry, the steps described here could be followed for developing representations of other geometries as well, through the replacement of correlations and the equivalent thermal resistance circuits.

The drawback of the choice of a simple solution is the loss of more complex behaviors of the heat exchange, such as stratification, which would require the presence of many derivatives for observing the temperature distribution through width and height of the outflow. However, even though important, such behaviors are not required for applications under which this model would be used, where only an approximated mean temperature provides enough information.

6. ACKNOWLEDGEMENTS

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8. RESPONSIBILITY NOTICE