EXPERIMENTAL DETERMINATION OF UNMANNED AIRCRAFT INERTIAL PROPERTIES

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Abstract. *This paper presents the methodology used in the experimental determination of the inertial properties of an unmanned aerial vehicle (UAV). This UAV is a prototype aircraft made of ply wood, balsa wood and aluminum, with a wingspan of 2,5m. A compound-pendulum is used to determine moments of inertia about transversal, longitudinal and vertical axes. The vertical location of the center of gravity of the aircraft is also determined. Experimental data are presented and discussed.*

Keywords*: Unmanned aerial vehicle, inertial properties, moments of inertia, experimental analysis, aircraft.*

1. INTRODUCTION

An Unmanned Aerial Vehicle – UAV – is a remotely controlled or autonomous aircraft, meaning that it is operated without a flight crew member on board. This class of aircraft is useful in situations where it is too dangerous or too expensive to use manned aircraft.

Since the beginning of 2005, the *Instituto Tecnológico de Aeronáutica* – ITA, São José dos Campos-SP, Brazil, has been developing an UAV, sponsored by *Financiadora de Estudos e Projetos* – FINEP, with the partnership of *Centro de Estudos Avançados do Recife* – CESAR – and *Companhia Hidro-Elétrica do São Francisco* - CHESF.

The main objective of this project is the development of an UAV capable of inspecting electrical transmission lines, with reliability. Nowadays, this inspection is made by helicopters, typically with one pilot and a technician filming or taking pictures.

The UAV requirements are:

- cruising speed: 120 km/h;
- capability to fly at 80 km/h up to 160 km/h;
- capability to fly at 1000 m above sea level;
- capability to take off and landing in rough fields, like grass or vicinal roads;
- landings and take-offs are not automatic, and will be conducted by humans, with remote control, like aero models;
- the UAV must carry a camera to film the power lines. In order to minimize image vibrations, the camera must be fixed near the center of gravity of the airplane;
- flight range in the preliminary project: 40 km;
- the power lines must be inspected (filmed) at horizontal distance of 25m and vertical distance of 30 m;
- the UAV must automatic follow the power lines using on-board computers (automatic pilot) and GPS technology.

In this UAV Project, a prototype is being developed, constructed and tested. The main objective of this project is to verify whether a small aircraft, subjected to gust loads, will be capable of filming with suitable quality the power lines. In this “proof of concept”, aspects like long range, durability or maintenance were much less important than low costs and rapid and easy construction. So, the airplane structure of the prototype is made of balsa wood and ply wood. Further versions will possibly be made of composite materials. Electronic hardware will be acquired and integrated on the airplane.

Girardi and Rizzi (2006, 2005a, 2005b) presented the methodology used to design the UAV, which main characteristics of the UAV are shown in Table 1.

Another platforms rather than airplane were considered (Girardi and Rizzi, 2005c), like helicopters or zeppelins, but the airplane seemed to be the most suitable option.

Position of center of gravity of the UAV and its moments of inertia are essential for analysis of performance and guidance. Analytical methods were used in first steps of UAV design, but the necessity of accuracy of these data was the motivation for the experimental determination of inertia parameters.

Table 1. Main characteristics of the UAV.

Parameter	
Wing area (S)	0.883 m^2
Wing span (b)	2.486 m
Wing chord (C_w)	0.355 m
Aileron chord (C_A)	0.071 m
Maximum take off weight (MTOW)	206 N
Limit positive load factor	4.4
Ultimate positive load factor	6.6
Limit negative load factor	-2.2
Ultimate negative load factor	-3.3

2. DETERMINATION OF CENTER OF GRAVITY

Figure 1 presents the aircraft body axes. The position of the center of gravity (CG) must be determined in both x and z coordinates. Of course, due to symmetry, the y-coordinate of CG is null.

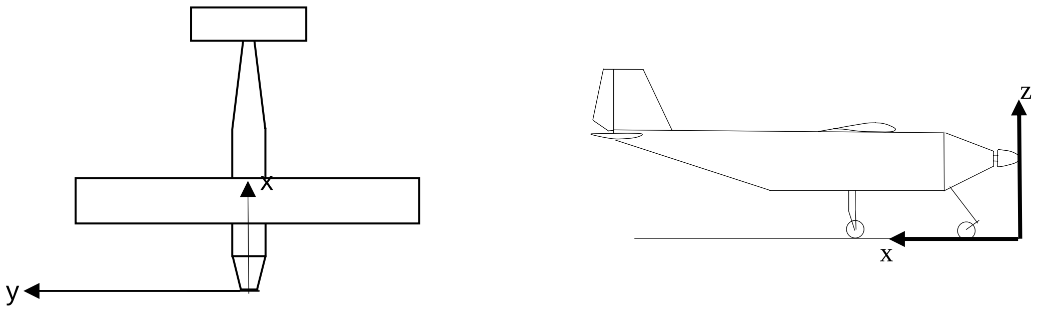

Figure 1 – UAV body axes.

2.1 Z-coordinate of CG

The vertical position of center of gravity, z_{CG}, can be determined by tilting the airplane, as suggested by Wolowicz and Yancey (1974). Figure 2 shows the swinging gear arranged to tilt the UAV.

A spirit level may be useful to guarantee that the aluminum frame is perfectly horizontal. Then, UAV must be placed on the frame, keeping the bars leveled, and meaning that the centers of gravity of UAV and swinging gear are vertically aligned.

Extra weight w is used to unbalance the system, tilting the assemblage around axis x and y (Figs. 3 and 4).

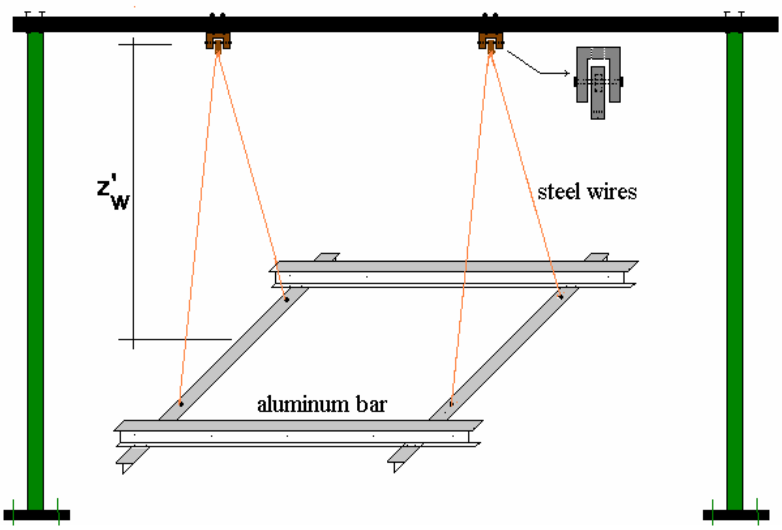

Figure 2 – Swinging gear.

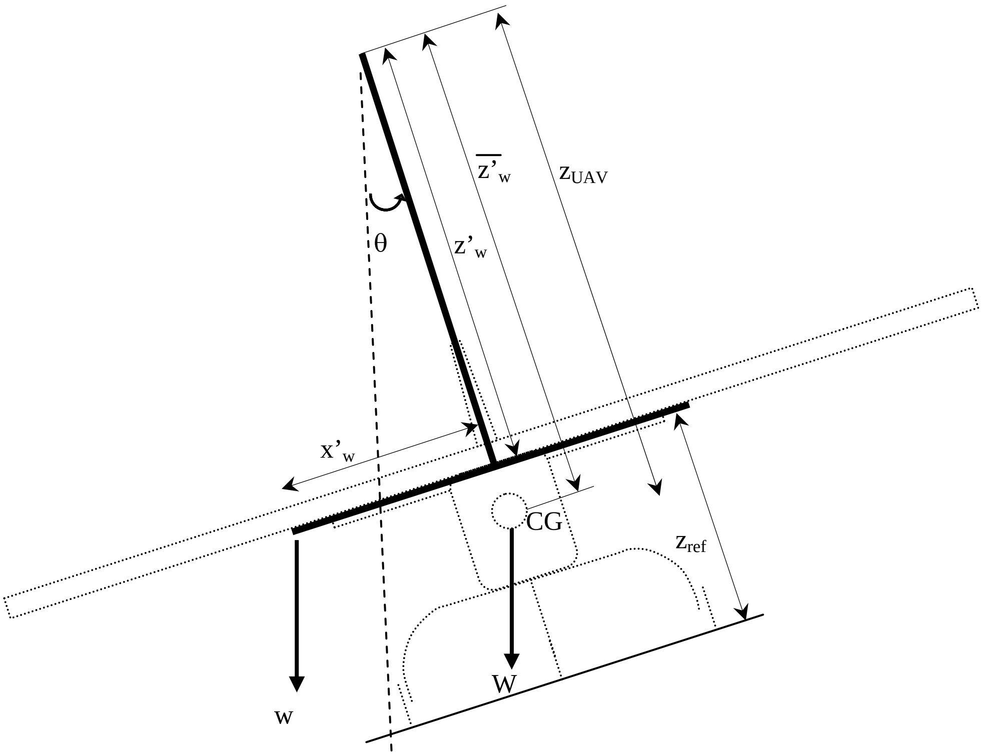

Figure 3 – UAV and swinging gear tilting about x axis.

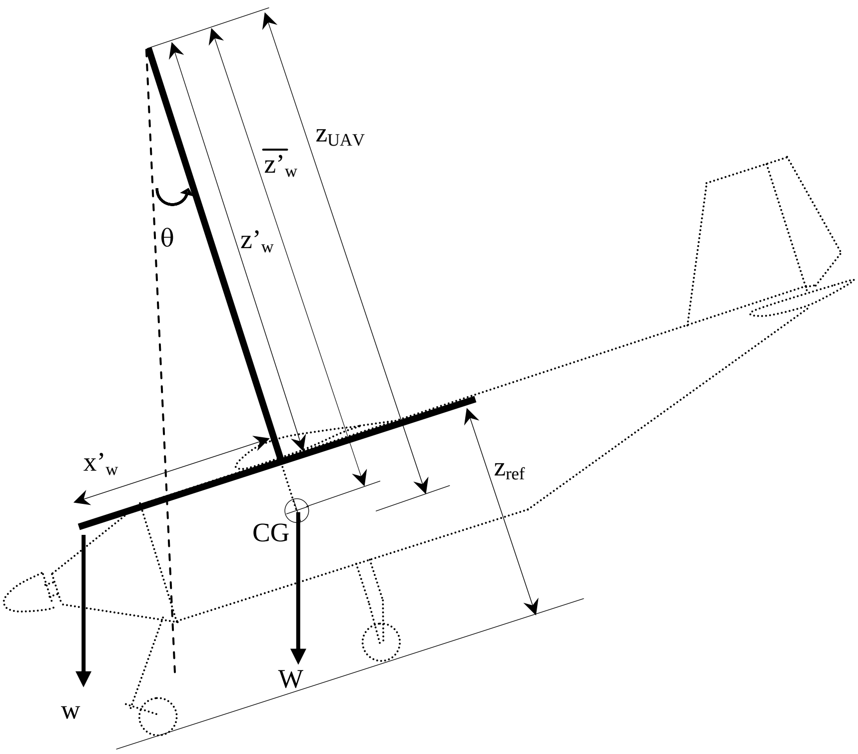

Figure 4 – UAV and swinging gear tilting about y axis.

From Figures 3 and 4, the moment equilibrium leads to:

$$W\bar{z}'\sin\theta = w(x'_w \cos\theta - z'_w \sin\theta) \tag{1}$$

where:

- CG is the center of gravity of the assemblage (UAV + swinging gear);
- W is the total weight (airplane plus swinging gear);
- w is the extra weight, used to tilt the UAV;
- $\bar{z}'$ is the vertical distance from pivot to the center of gravity of the assemblage (UAV + swinging gear);
- x'_w is the horizontal distance from extra weight (w) to CG;
- θ is the tilt angle;
- z'_w is the vertical distance from pivot to the reference (CG of swinging gear);

and the vertical position of CG of the assemblage is given by:

$$\bar{z}' = \frac{w}{W}\left(\frac{x'_w}{tg\theta} - z'_w\right) \tag{2}$$

Finally, the desired position of the center of gravity of the UAV, z_{UAV}, must be isolated. It can be calculated by:

$$z_{UAV} = \frac{\bar{z}'W - z_{SG}W_{SG}}{W_{UAV}} \tag{3}$$

The tilt angle θ can be measured with a laser ray emitted by a simple laser pointer device. This visible ray can be projected onto a white wall or board. Figure 5 illustrates the geometry of the tilted set.

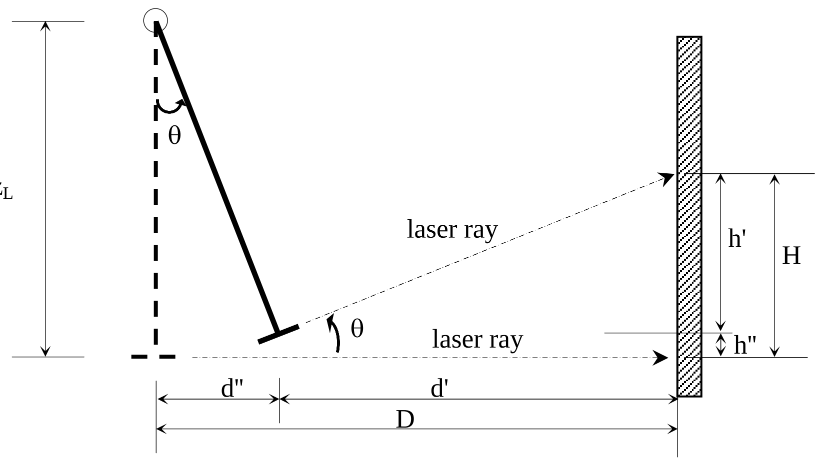

Figure 5 – Experimental determination of tilt angle θ

From Fig.5:

$$tg\theta = \frac{h'}{d'} = \frac{H - h''}{D - d''} = \frac{H - (z_L - z_L\cos\theta)}{D - z_L sen\theta} \tag{4}$$

where:

- z_L is the vertical distance from pivot to laser ray;
- H is the total vertical deflection of laser ray projection;

- D is the horizontal distance from pivot to the white wall .

With the help of Mathematica©, a well-known computing software, the solutions of (4) are given by:

$$\theta = \pm \arccos\left(\frac{-Hz_L + z_L^2 \mp D\sqrt{D^2 + H^2 - 2Hz_L}}{D^2 + H^2 - 2Hz_L + z_L^2} \right) \tag{5}$$

2.2. Experimental results of Z_{CG}

An aluminum frame was built, as proposed in Fig. 2, to perform the inertia tests. This frame is suspended by four steel wires, attached to two hinges. Total weight of this swinging gear is W_{SG}=2.953 kgf. This arrangement can be seen in Fig. 6, where a laser pen device can be noted (fixed at the frame), projecting a visible shot over the white board.

Figure 6 – Swinging gear with laser device and load.

UAV was arranged in two different positions, tilting around both x and y axes (Fig. 7). Table 2 shows geometric data for both arrangements.

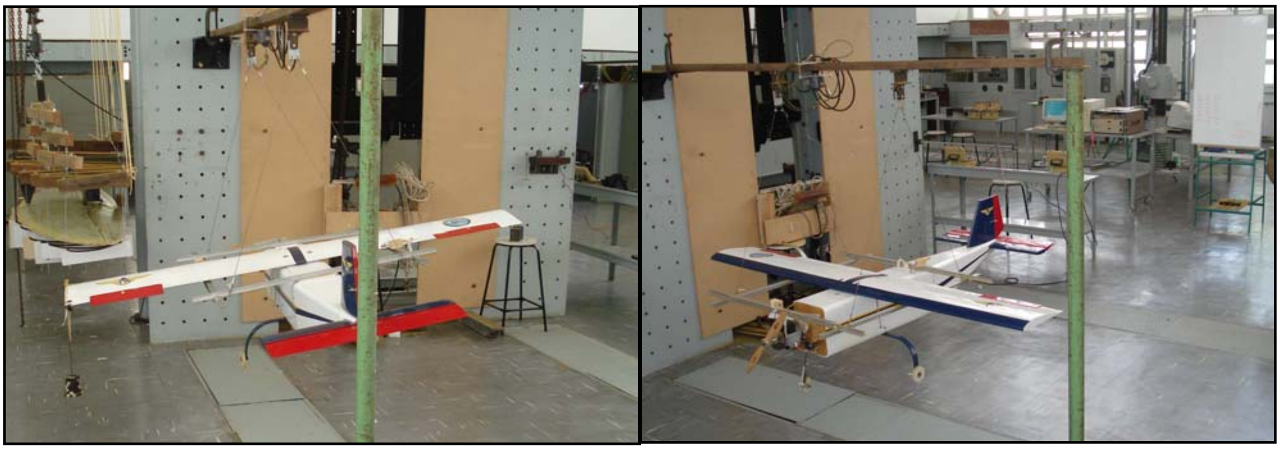

Figure 7 – UAV tilting around x and y axes, respectively.

Table 2 - Experimental data for determination of vertical position of CG

Geometry	Tilting about x axis	Tilting about y axis	Unit
W_{UAV}	21.001	21.001	kgf
W_{SG}	2.953	2.953	kgf
W	23.953	23.953	kgf
z_{sg}	1094.0	1094.0	mm
z_{ref}	550.0	550.0	mm
z_L	1164.0	1164.0	mm
D	5917.0	5917.0	mm
x'_w	1235.0	496.0	mm
z'_w	1094.0	1094.0	mm

Table 3 - Vertical position of the center of gravity - tilting about x axis

	1	2	3	4	5	6	7	8	9
H (mm)	288.2	385.0	478.0	577.0	666.0	755.5	816.5	882.0	1036.0
W (kgf)	1.1693	1.5884	2.0059	2.4667	2.8943	3.3405	3.6499	3.9930	4.8405
θ (rad)	0.049	0.065	0.081	0.098	0.113	0.129	0.139	0.150	0.176
θ (degree)	2.80	3.75	4.66	5.62	6.49	7.37	7.96	8.60	10.10
$\bar{z}'$ (mm)	1178.4	1178.0	1178.4	1178.9	1178.8	1179.3	1178.4	1178.3	1179.5
z_{UAV} (mm)	1190.3	1189.8	1190.2	1190.9	1190.7	1191.2	1190.2	1190.2	1191.6

Table 4 - Vertical position of the center of gravity - tilting about y axis

	1	2	3	4	5	6	7	8
H (mm)	115.8	154.5	192.0	232.5	269.0	305.5	330.5	358.0
w (kgf)	1.1693	1.5884	2.0059	2.4667	2.8943	3.3405	3.6499	3.9930
θ (rad)	0.020	0.026	0.033	0.039	0.046	0.052	0.056	0.061
θ (degree)	1.12	1.50	1.86	2.26	2.61	2.97	3.21	3.48
$\bar{z}'$ (mm)	1181.4	1183.9	1184.3	1182.2	1180.2	1180.3	1178.9	1176.0
z_{UAV} (mm)	1193.7	1196.5	1197.0	1194.6	1192.3	1192.5	1190.9	1187.6

Table 3 shows results when UAV was tilted around x axis, for 9 different weights. The mean value of z_{UAV}, from Table 3, is 1190.6mm, and the standard deviation is 0.569. Taking into account the geometry of UAV and its assemblage on the swinging gear, the vertical position of CG is:

$$z_{CG} = z'_w + z_{ref} - z_{UAV} = 1094.0 + 550.0 - 1190.6 = 453.4 \text{ mm} \quad (6)$$

The geometry and numerical results of tilting UAV around y axis are shown in Table 4. The mean value of z_{UAV} is 1193.1 mm, and the standard deviation is 3.077 mm. Taking account for the geometry of UAV and its assemblage on the swinging gear, it means that the vertical position of CG is:.

$$z_{CG} = z'_w + z_{ref} - z_{UAV} = 450.9 \text{ mm} \quad (7)$$

The difference between values found in (6) and (7) is 2.5 mm, less than standard deviation.

2.3 X-coordinate of CG

The x-coordinate of the center of gravity is very simply to determine experimentally. Figure 8 illustrates how 2 scales can be used.

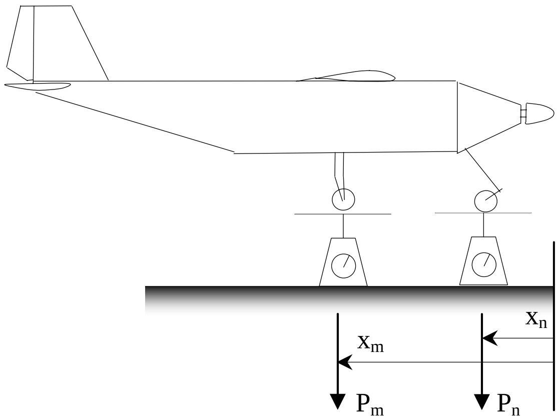

Figure 8 – Determination of x-coordinate of CG

Then, the x-coordinate of CG is given by:

$$x_{CG} = \frac{x_m P_m + x_n P_n}{P_m + P_n} \tag{8}$$

Alternatively, the arrangement used to determinate z_{CG} can be used. If both UAV and the frame are horizontal, x_{CG} can be simply determined by measuring the horizontal distance from the origin of x axis (the nose of the UAV) to the pivot.

3. DETERMINATION OF THE MOMENTS OF INERCIA

The method used for determination of moments of inertia involves swinging the airplane as a pendulum, as suggested by Miller (1930).

The moment of inertia of the UAV, I_{UAV}, about axis x or y, can be found by oscillating the swinging gear with the UAV, and then measuring its period T. The same apparatus described in previous section can be used. The moment of inertia of the swinging gear alone must be subtracted of total moment of inertia, then the period of oscillation of swinging gear alone, T_{SG}, must be measured too. Also, the extra moment of inertia of UAV due to displacement of its center of gravity must be taken into account. So, the expression of moment of inertia of UAV can be summarized by:

$$I_{UAV} = \frac{W\,T^2\,\bar{z}'}{4\pi^2} - \frac{W_{SG}\;T_{SG}^2\;z'_w}{4\pi^2} - \frac{W_{UAV}\;z_{UAV}}{g} \tag{9}$$

where g is acceleration of gravity, assumed as 9.81m/s^2.

For the moment of inertia about a vertical axis, I_{zz}, a bifilar pendulum is used (Fig.9), with two vertical fibers of length L, separated by a horizontal distance a. This moment of inertia can be expressed by:

$$I_{zz} = \frac{W\,T^2\,a^2}{16\pi^2 L} - \frac{W_{SG}\;T_{SG}^2\;a^2}{16\pi^2 L} \tag{10}$$

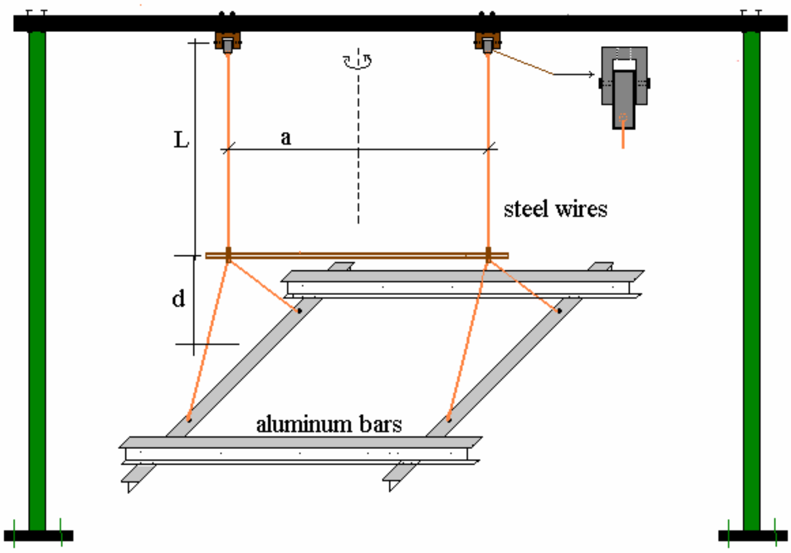

Figure 9 – Bifilar torsion pendulum for determination of I_{zz}

Formulas (9) and (10) are valid for small amplitudes of oscillation, since sinθ = tanθ = θ is assumed (where 2θ is the angle of oscillation).

3.1 Experimental results of moments of inertia

Periods required on Equations 9 and 10 were determined with 50 oscillations of the pendulum.

Several measurements of elapsed time for 50 oscillations were taken, and Tables 5 and 6 show mean values. For determination of I_{xx} and I_{yy}, the swinging gear used was the same used for determination of CG (Figure 7).

Table 5 –Determination of I_{xx}

Distances (m)	$\overline{z'}$=1.178 ; z'_w=1.094 ; z_{UAV}=1.190	
Case	Swinging gear only	UAV
Elapsed time (s)	114.0	116.0
Period (s)	2.28	2.32
I_{xx} (kg.m^2)		3.842

Table 6 –Determination of I_{yy}

Distances (m)	$\overline{z'}$=1.182 ; z'_w=1.091 ; z_{UAV}=1.194	
Case	Swinging gear only	UAV
Elapsed time (s)	113.4	119.8
Period (s)	2.27	2.40
I_{xx} (kg.m^2)		6.302

The swinging gear for determination of I_{zz} weights 3.6518 kgf. An aluminum bar of length 2.2895m, and rectangular cross section (0.04761 m x 0.04757 m), weighing 14.400 kgf, was used to check the methodology of determination of I_{zz}. Figure 10 shows both UAV and aluminum bar arrangements. Two different lengths of b were used. Table 7 summarizes these results.

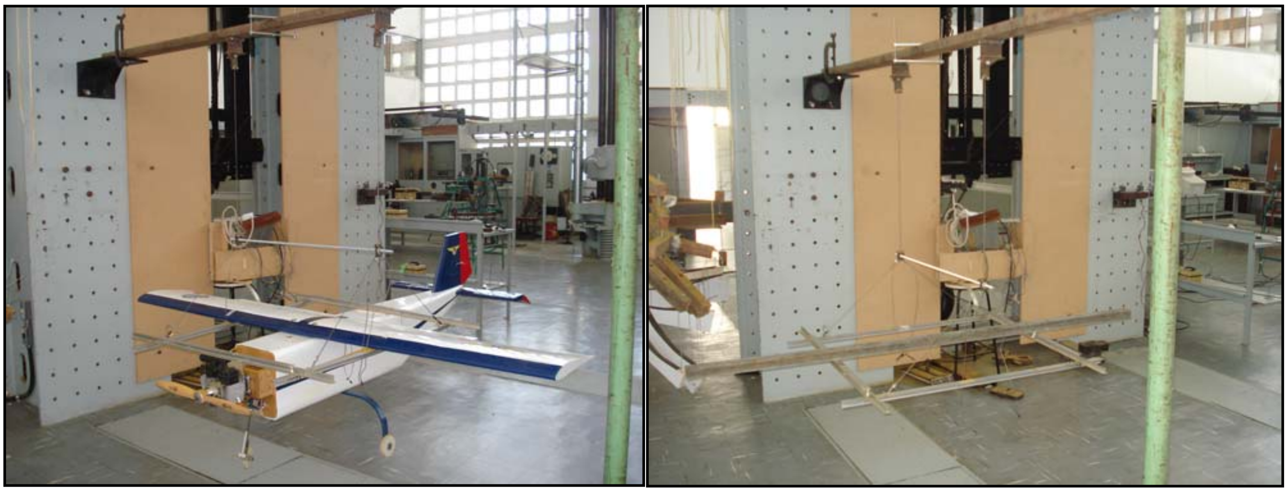

Figure 10 – UAV and bar determination of I_{zz}

Table 7 – Determination of I_{zz}

Geometry	d=0.355m			d=0.139m		
Case	Swinging gear only	Bar	UAV	Swinging gear only	Bar	UAV
Elapsed time for 50 cycle (s)	121.8	135.2	111.4	109.6	133.3	110.6
	122.2	136.0	112.1	110.7	133.3	110.2
	121.9	136.8	111.1	-	-	110.1
	-	-	112.2	-	-	110.0
Mean Period (s)	2.44	2.72	2.23	2.20	2.67	2.20
I_{zz} (kg.m^2)	1.28	6.57	5.95	1.04	6.49	5.99

Knowing that, for the aluminum bar, theoretical I_{zz} equals to 6.287 kg.m^2, errors found in the tests are 4.4% and 3.3%, for longer swinging gear (d=0.355m) and the shorter one, respectively. Longer vertical distances d may cause secondary oscillations, so this length should be kept as short as possible.

4. CONCLUSIONS

Experimental techniques were used to measure inertia properties of an UAV. These experiments showed to be very simple and cheap to be performed. The swinging gear can be reused in another small aircrafts, like UAV's and air models.

Some care must be taken to avoid errors. The apparatus must be free of friction, and the horizontal frame must be stiff enough to eliminate significant bending and torsion. The measurement of tilt angle is very susceptible to gross errors, and the determination of vertical position of center of gravity is very sensible to this angle. Good results were found when the laser device was fixed at the frame, at center point of the horizontal bar, the one parallel to the axis of tilting.

Secondary oscillations on determination of moments of inertia must be eliminated. The accuracy of the center of gravity is essential to the precision of moments of inertia. Time measurements were taken with a simple chronometer. To avoid gross errors, fifty oscillations were performed to minimize human factors. Electronic chronometer may be used.

5. ACKNOWLEDGEMENTS

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