FAULT-TOLERANT FLIGHT CONTROL SYSTEM USING MODEL PREDICTIVE CONTROL

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Abstract. *This work presents a fault-tolerant flight control system using model predictive control (MPC). The proposed technique, named feasible target-tracking MPC, filters the reference demand to guarantee feasibility of the constrained optimization. This architecture is also capable of redistributing, in a stable manner, the control efforts among healthy actuators, respecting their limitations. A trajectory tracking system based on the proposed fault-tolerant model predictive controller is demonstrated using the ground simulator of the VFW-614 ATTAS (Advanced Technologies Testing Aircraft System), showing feasibility and adequate performance.*

Keywords: *Model predictive control, Fault-tolerant control, Domain of attraction, Target-tracking*

1. INTRODUCTION

Model predictive control (MPC) is widely used today as a strategy to control plants with input and output constraints, which also has encouraged its application to design fault-tolerant controllers, since actuator faults reflect into new constraints and then the internal optimizer tries to find an optimal solution considering the new limitation. Maciejowski and Jones (2003), Kale and Chipperfield (2005) and Miksch *et al.* (2008), among others, have investigated the benefits and disadvantages of MPC for fault-tolerant control. However, most of the implementations make use of a cost function with two different horizons, namely *prediction* and *control* horizons. Those schemes have no guarantee of closed-loop stability (Mayne *et al.*, 2000) and introduce additional complexity during the design phase.

Recently, Almeida and Leissling (2009) proposed a fault-tolerant tracking MPC with infinite prediction horizon and a translated terminal invariant set to guarantee stability. It is already known (Limon *et al.*, 2005) that terminal sets significantly reduce the domain of attraction of the controller, thus large control horizons are required to maintain feasibility for a given state vector. Such large horizons would be implementable for controlled processes with slow dynamics. Unfortunately, this is not the case for flight control systems, where the dynamical modes are normally faster than those controlled by MPC strategies in the process industry. Also, the on-board available computational resources must be shared with several functionalities of the complete guidance and control system.

Focusing on the design of fault-tolerant flight control systems, this work presents a new technique denominated feasible target-tracking MPC, which demands shorter control horizons than the current MPC formulations. The key concept is to filter the reference demand to guarantee the feasibility of the constrained regulator. The modification of the demand is done by a steady-state target optimizer, considering the current state and disturbance acting on the system. A similar procedure of demand filtering can be found in Limon *et al.* (2008), but here the sets of constraints on state and input do not necessarily contain the origin, allowing the application of the proposed technique in various scenarios of faults. Also, the controller incorporates a state and disturbance observer to track a desired reference without offset error.

This paper is organized as follows. In the next section, the proposed feasible-target tracking MPC is presented. Section 3 presents a trajectory-tracking system with the lateral autopilot designed through the technique of Section 2. The implementation of the complete system into the ATTAS ground simulator, and the correspondent simulation results, are presented and discussed in Section 4.

2. FEASIBLE TARGET-TRACKING MODEL PREDICTIVE CONTROL

This section presents a new model predictive control technique for tracking a desired reference. Based on the values of the state and disturbance vectors, feasible target state and control vectors are computed and utilized by the infinite horizon MPC regulator. The weighting matrices of the regulator are selected according to the implicit model following technique. This choice provides that the eigenstructure of the closed-loop system will be kept close (in a least-squares sense) to a specified model even with actuator faults, provided that enough analytical redundancy exists in the system. Also, a linear observer is proposed to provide offset-free tracking of a time-varying reference, which improves significantly the robust performance of the controller.

Figure 1 shows the overall structure of the proposed solution. The discrete controller (circumvented by the dotted line) has three main functionalities: a feasible target calculation, the MPC optimizer, and a linear observer. It is assumed that a fault detection and isolation system (FDI) provides to both target calculation and MPC schemes correct information about the status of the actuators.

The feasible target calculation subsystem computes the state and control vectors ($\mathbf{x}_{ss}$ and $\mathbf{u}_{ss}$) at the steady-state,

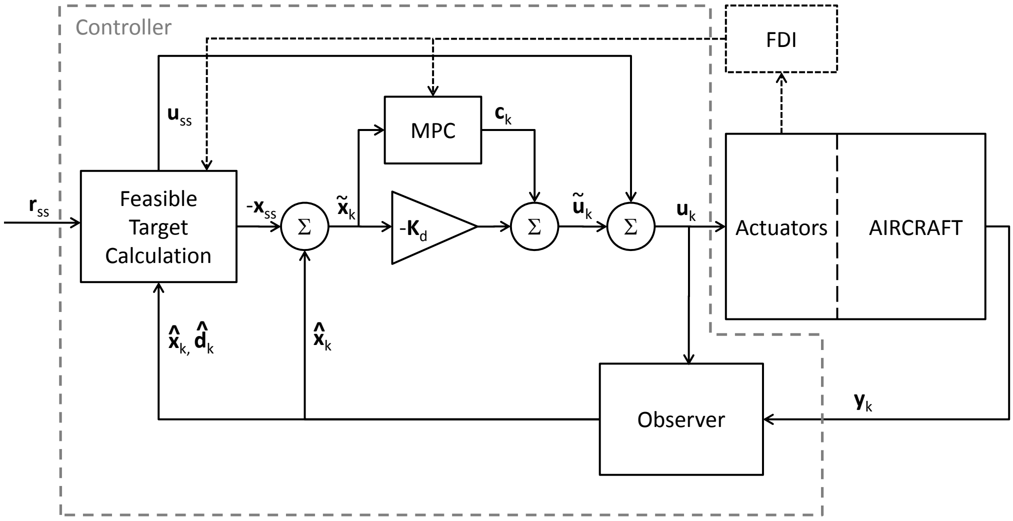


Figure 1. Overall structure of the proposed feasible target-tracking MPC

which are required to give offset-free tracking of the reference signal $\mathbf{r}_{ss}$. This calculation considers the limitations of the actuators, thus comprising a constrained problem. Here is the main contribution of this work: a conversion of those limitations into a new set of constraints is proposed, which guarantees that - based on the computed vectors $\mathbf{x}_{ss}$ and $\mathbf{u}_{ss}$ - the MPC regulator reaches a feasible solution.

With the calculated steady-state values, the estimated state vector $\hat{\mathbf{x}}_k$ is subtracted of $\mathbf{x}_{ss}$ to convert the tracking problem into a regulation over the desired steady-state condition. In turn, the regulator control law is the sum of two contributions: a linear part with static linear feedback gain $\mathbf{K}_d$ and a nonlinear correction $\mathbf{c}_k$ calculated by the MPC system. The MPC is designed in a way that only generates corrections if $\hat{\mathbf{x}}_k$ is such that the linear control part pushes the actuators and/or aircraft against the limitations. This means also that, in the event of an actuator fault, the nonlinear correction $\mathbf{c}_k$ redistributes the control effort among the available actuators. A linear, unconstrained observer is employed in the proposed scheme to provide proper estimates of the disturbance $\hat{\mathbf{d}}_k$ and state $\hat{\mathbf{x}}_k$. The estimation of the disturbance is a crucial element to avoid offset when tracking a desired reference. Here, not only exogenous elements (e.g. turbulence) but also unmodeled actuator and aircraft dynamics are considered as disturbances.

2.1 Feasible target calculation

Let the discrete-time state-space model be defined by

$$\begin{aligned}
\mathbf{x}_{k+1} &= \mathbf{\Phi}\mathbf{x}_k + \mathbf{\Gamma}\mathbf{u}_k + \mathbf{\Gamma}_d\mathbf{d}_k \\
\mathbf{d}_{k+1} &= \mathbf{d}_k \\
\mathbf{y}_k &= \mathbf{E}\mathbf{x}_k \\
\mathbf{z}_k &= \mathbf{H}\mathbf{y}_k
\end{aligned} \tag{1}$$

where $\mathbf{x}_k \in \mathbb{R}^n$ is the state vector, $\mathbf{u}_k \in \mathbb{R}^m$ is the control vector, $\mathbf{y}_k \in \mathbb{R}^p$ is the vector of observed variables, $\mathbf{z}_k \in \mathbb{R}^q$, $m \geq q$ is the vector of controlled variables and $\mathbf{d}_k \in \mathbb{R}^q$ is the vector of disturbances. The motivation of incorporating a disturbance vector is to consider mismatches between plant and nominal model, as well as external disturbances acting on the plant. The pair $(\mathbf{E}, \mathbf{\Phi})$ is assumed to be detectable with $\mathbf{E}$ full row rank. Also, the disturbance vector is assumed to be estimated by a proper observer (to be introduced later).

The disturbance model chosen is a simple integrator. If desired, one can choose different dynamics for the disturbance, as done in Pannocchia and Rawlings (2003). The disturbance matrix $\mathbf{\Gamma}_d$ must be chosen by the designer and thus introduces a degree of flexibility in the design. Its choice may be motivated by an actual disturbance acting on the system or may let be free as a design parameter. Since the disturbance is not measured, the choice of the disturbance matrix may be a significant challenge.

Thus, the objective of the control system is to asymptotically eliminate the tracking error given a reference signal $\mathbf{r}_{ss}$, that is

$$\mathbf{z}_k \xrightarrow[k\to\infty]{} \mathbf{r}_{ss} \tag{2}$$

in the presence of disturbance $\mathbf{d}_k$ and constraints in the state and control vectors $\mathbf{x}_k \in \mathbf{X}, \mathbf{u}_k \in \mathbf{U}$, where $\mathbf{X}$ and $\mathbf{U}$ are closed, bounded and convex sets expressed by linear inequalities. This problem corresponds to finding the new equilibrium point of the plant in steady-state, which turns into the determination of the steady-state target vectors $\mathbf{x}_{ss}\left(\mathbf{r}_{ss},\mathbf{d}_k\right)$ and $\mathbf{u}_{ss}\left(\mathbf{r}_{ss},\mathbf{d}_k\right)$.

In the context of fault-tolerant flight control, it is often the case that constraints are active in steady-state. This may happen when actuators jam at a non-zero position or the disturbance $\mathbf{d}_k$ is large enough to push states and controls against the constraints. Considering that $m \geq q$, the following constrained target calculation problem is solved to obtain $\mathbf{x}_{ss}$ and $\mathbf{u}_{ss}$

$$\begin{aligned}
&\min_{\mathbf{x}_{ss},\mathbf{u}_{ss}} J\left(\mathbf{r}_{ss},\mathbf{d}_k\right) = \left(\mathbf{HEx}_{ss}-\mathbf{r}_{ss}\right)^T\mathbf{Q}_{ss}\left(\mathbf{HEx}_{ss}-\mathbf{r}_{ss}\right)+\mathbf{u}_{ss}^T\mathbf{R}_{ss}\mathbf{u}_{ss}\\
&\qquad\text{subject to:}\\
&\qquad\left(\mathbf{\Phi}-\mathbf{I}\right)\mathbf{x}_{ss}+\mathbf{\Gamma}\mathbf{u}_{ss}+\mathbf{\Gamma}_d\mathbf{d}_k = 0\\
&\qquad\qquad\qquad\mathbf{u}_{ss}\in\mathbf{U}\\
&\qquad\qquad\qquad\mathbf{x}_{ss}\in\mathbf{X}
\end{aligned} \tag{3}$$

where $\mathbf{Q}_{ss}$ and $\mathbf{R}_{ss}$ are weighting matrices. This corresponds to a quadratic programming problem, whose feasibility is guaranteed by the penalization of the difference between the desired ($\mathbf{r}_{ss}$) and the reachable ($\mathbf{z}_{ss} = \mathbf{HEx}_{ss}$) target. If the constraints are too stringent, the reachable target will be near to the desired target in a least-squares sense. The second term of the cost function penalizes the control vector in order to produce a small $\mathbf{u}_{ss}$. Nevertheless, feasibility of this constrained problem does not guarantee feasibility of the constrained MPC regulator, because the estimated state and disturbance vectors are not taken into account. For given $\hat{\mathbf{x}}_k$ and $\hat{\mathbf{d}}_k$, there is an allowable set of demands that would drive the constrained regulator to a feasible zone, reflecting into smaller admissible sets of $\mathbf{x}_{ss}$ and $\mathbf{u}_{ss}$.

The starting point to determine those sets is the proposed control law predicted for N steps

$$\begin{aligned}
\mathbf{u}_j &= -\mathbf{K}_d\left(\mathbf{x}_j-\mathbf{x}_{ss}\right)+\mathbf{u}_{ss}+\mathbf{c}_j \qquad j=0,\ldots,N-1\\
\mathbf{u}_j &= -\mathbf{K}_d\left(\mathbf{x}_j-\mathbf{x}_{ss}\right)+\mathbf{u}_{ss} \qquad\qquad j\geq N
\end{aligned} \tag{4}$$

where $\mathbf{x}_0 = \hat{\mathbf{x}}_k$. The MPC regulator calculates the contribution $\mathbf{c}_j$ over N steps. It is assumed that after N steps the constraints are no longer active, which is equivalent to saying that the state vector at $j = N$ is inside an invariant set $O_\infty\left(\mathbf{x}_{ss},\mathbf{u}_{ss}\right)$.

Therefore, assuming that $\mathbf{x}_{ss}$ and $\mathbf{u}_{ss}$ keep constant for a given reference, it will be convenient to extend the state-space vector as $\begin{bmatrix}\mathbf{x}_j^T & \mathbf{x}_{ss}^T & \mathbf{u}_{ss}^T & \mathbf{d}_j^T\end{bmatrix}^T$, leading to the following extended dynamical model

$$\begin{bmatrix}\mathbf{x}_{j+1}\\ \mathbf{x}_{ss}\\ \mathbf{u}_{ss}\\ \mathbf{d}_{j+1}\end{bmatrix} = \begin{bmatrix}\mathbf{\Phi}-\mathbf{\Gamma}\mathbf{K}_d & \mathbf{\Gamma}\mathbf{K}_d & \mathbf{\Gamma} & \mathbf{\Gamma}_d\\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{x}_j\\ \mathbf{x}_{ss}\\ \mathbf{u}_{ss}\\ \mathbf{d}_j\end{bmatrix}+\begin{bmatrix}\mathbf{\Gamma}\\ \mathbf{0}\\ \mathbf{0}\\ \mathbf{0}\end{bmatrix}\mathbf{c}_j \tag{5}$$

The computation of the invariant set $O_\infty\left(\mathbf{x}_{ss},\mathbf{u}_{ss}\right)$ requires the correct formulation of the constraints that are imposed on $\begin{bmatrix}\mathbf{x}_j^T & \mathbf{x}_{ss}^T & \mathbf{u}_{ss}^T & \mathbf{d}_j^T\end{bmatrix}^T$. It is clear that state, input and disturbance constraints can be expressed in terms of linear inequalities:

$$\begin{aligned}
\mathbf{C}_c\mathbf{x}_j &\leq \mathbf{x}_{max}\\
-\mathbf{C}_c\mathbf{x}_j &\leq -\mathbf{x}_{min}\\
\mathbf{C}_c\mathbf{x}_{ss} &\leq \mathbf{x}_{max}\\
-\mathbf{C}_c\mathbf{x}_{ss} &\leq -\mathbf{x}_{min}\\
\mathbf{u}_{ss} &\leq \mathbf{u}_{max}\\
-\mathbf{u}_{ss} &\leq -\mathbf{u}_{min}\\
\mathbf{d}_j &\leq \mathbf{d}_{max}\\
-\mathbf{d}_j &\leq -\mathbf{d}_{min}\\
-\mathbf{K}_d\mathbf{x}_j+\mathbf{K}_d\mathbf{x}_{ss}+\mathbf{u}_{ss} &\leq \mathbf{u}_{max}\\
\mathbf{K}_d\mathbf{x}_j-\mathbf{K}_d\mathbf{x}_{ss}-\mathbf{u}_{ss} &\leq -\mathbf{u}_{min}
\end{aligned} \tag{6}$$

where $\mathbf{x}_{max}$, $\mathbf{x}_{min}$, $\mathbf{u}_{max}$, $\mathbf{u}_{min}$, $\mathbf{d}_{max}$ and $\mathbf{d}_{min}$ are the bounds on state, control and disturbance vectors, respectively.

Hence, the set $O_\infty\left(\mathbf{x}_{ss},\mathbf{u}_{ss}\right)$ can be constructed using the techniques described in Gilbert and Tan (1991), considering the extended dynamical system given by Eq. (5) and constraints Eq. (6). From $O_\infty\left(\mathbf{x}_{ss},\mathbf{u}_{ss}\right)$, one can compute the polyhedral set $\mathbf{X}_N$ of extended states that can be steered to $O_\infty\left(\mathbf{x}_{ss},\mathbf{u}_{ss}\right)$ by N control steps $\mathbf{C}_N = \{\mathbf{c}_j\}_{j=0}^{N-1}$ while respecting the constraints. This is done by recursion, substituting the dynamical system Eq. (5) into the constraints Eq. (6) for $j = 0, \cdots, N-1$, and then projecting the resulting convex set onto the subspace spanned by $\begin{bmatrix} \mathbf{x}_k^T & \mathbf{x}_{ss}^T & \mathbf{u}_{ss}^T & \mathbf{d}_k^T \end{bmatrix}^T$. The projected polyhedral set $\mathbf{X}_N$ assumes the form

$$\mathbf{X}_N = \{(\mathbf{x}_k, \mathbf{x}_{ss}, \mathbf{u}_{ss}, \mathbf{d}_k) \,|\mathbf{M}_x \mathbf{x}_k + \mathbf{M}_{x_{ss}} \mathbf{x}_{ss} + \mathbf{M}_{u_{ss}} \mathbf{u}_{ss} + \mathbf{M}_d \mathbf{d}_k \leq \mathbf{k}_N\} \tag{7}$$

Finally, the set $\mathbf{X}_{ss_N}$ of admissible steady-state values, which assures feasibility of the MPC regulator, is computed, at each sampling time, through substitution of $\hat{\mathbf{x}}_k$ and $\hat{\mathbf{d}}_k$ into $\mathbf{X}_N$. This operation is often called *slice* of a polyhedron. Thus, the feasible target values of $\mathbf{x}_{ss}$ and $\mathbf{u}_{ss}$ are obtained through the solution of the quadratic programming problem

$$\begin{aligned} \min_{\mathbf{x}_{ss}, \mathbf{u}_{ss}} J\left(\mathbf{r}_{ss}, \hat{\mathbf{x}}_k, \hat{\mathbf{d}}_k\right) &= \left(\mathbf{H}\mathbf{E}\mathbf{x}_{ss} - \mathbf{r}_{ss}\right)^T \mathbf{Q}_{ss} \left(\mathbf{H}\mathbf{E}\mathbf{x}_{ss} - \mathbf{r}_{ss}\right) + \mathbf{u}_{ss}^T \mathbf{R}_{ss} \mathbf{u}_{ss} \\ &\text{subject to:} \\ &\left(\mathbf{\Phi} - \mathbf{I}\right) \mathbf{x}_{ss} + \mathbf{\Gamma} \mathbf{u}_{ss} + \mathbf{\Gamma}_d \mathbf{d}_k = 0 \\ &\left(\mathbf{x}_{ss}, \mathbf{u}_{ss}\right) \in \mathbf{X}_{ss_N} \end{aligned} \tag{8}$$

It should be noted that $\mathbf{X}_{ss_N} \subseteq \mathbf{X}_{ss_{N+1}}$, because larger control horizons mean more discrete steps to steer the extended state-space into the invariant set $O_\infty\left(\mathbf{x}_{ss}, \mathbf{u}_{ss}\right)$. This is equivalent to say that, for a given pair $\left(\hat{\mathbf{x}}_k, \hat{\mathbf{d}}_k\right)$, the set of admissible steady-state references $\mathbf{z}_{ss}$ is wider with larger control horizons.

2.2 Constrained implicit model following regulator

The implicit model following form of the MPC regulator (Almeida and Leissling, 2009) was chosen because it provides proper dynamic control allocation in the case of one or more actuators reaching the saturation, maintaining the transient response as close as possible to the specified reference model.

The objective of the regulator is to drive the state vector to the origin, given $\mathbf{x}_0 = \mathbf{x}_k$. Converting this to a tracking control problem is done simply by translating the origins of state and control vectors to the desired state and control reference values $\mathbf{x}_{ss}$ and $\mathbf{u}_{ss}$. Defining $\tilde{\mathbf{x}}_j = \mathbf{x}_j - \mathbf{x}_{ss}$ and $\tilde{\mathbf{u}}_j = \mathbf{u}_j - \mathbf{u}_{ss}$, the regulator problem is converted to a tracker replacing $\mathbf{x}$ and $\mathbf{u}$ by $\tilde{\mathbf{x}}$ and $\tilde{\mathbf{u}}$, respectively. The translation also applies to the initial state $\mathbf{x}_0$, admissible sets $\mathbf{U}$, $\mathbf{X}$ and the invariant set O_∞. It should be realized that this translation cancels the influence of the disturbance on the transient response, because $\mathbf{d}_k = \mathbf{d}_{k+1} = \hat{\mathbf{d}}_k$ and then $\tilde{\mathbf{d}}_k = 0$. Hence, the nominal system for regulation is given by

$$\tilde{\mathbf{x}}_{k+1} = \mathbf{\Phi}\tilde{\mathbf{x}}_k + \mathbf{\Gamma}\tilde{\mathbf{u}}_k \tag{9}$$

The idea of implicit model following is to modify the continuous-time LQR cost function, penalizing the deviation of the system output from a certain reference model output. Let the continuous-time representation of Eq. (9) be defined by

$$\begin{aligned} \dot{\tilde{\mathbf{x}}} &= \mathbf{A}\tilde{\mathbf{x}} + \mathbf{B}\tilde{\mathbf{u}} \\ \mathbf{w} &= \mathbf{C}\tilde{\mathbf{x}} \end{aligned} \tag{10}$$

Suppose that the performance output $\mathbf{w}$ in Eq. (10) is required to follow the transient behavior of the autonomous model

$$\dot{\mathbf{w}}_m = \mathbf{A}_m \mathbf{w}_m \tag{11}$$

The matrix $\mathbf{A}_m$ has the reference eigenstructure of the regulator. When the control objective is met, the performance output $\mathbf{w}$ will satisfy Eq. (11). One can define an error by

$$\mathbf{e} = \dot{\mathbf{w}} - \mathbf{A}_m \mathbf{w} \tag{12}$$

and a cost function by

$$J = \int_0^\infty \left(\mathbf{e}^T \mathbf{Q} \mathbf{e} + \tilde{\mathbf{u}}^T \mathbf{R} \tilde{\mathbf{u}}\right) dt \tag{13}$$

Since $\dot{\mathbf{w}} = \mathbf{C}\dot{\tilde{\mathbf{x}}} = \mathbf{C}\mathbf{A}\tilde{\mathbf{x}} + \mathbf{C}\mathbf{B}\tilde{\mathbf{u}}$, the cost function becomes

$$J = \int_0^\infty \left(\tilde{\mathbf{x}}^T \mathbf{Q}_m \tilde{\mathbf{x}} + 2\tilde{\mathbf{x}}^T \mathbf{W}_m \tilde{\mathbf{u}} + \tilde{\mathbf{u}}^T \mathbf{R}_m \tilde{\mathbf{u}}\right) dt \tag{14}$$

where

$$\begin{aligned} \mathbf{Q}_m &= (\mathbf{CA} - \mathbf{A}_m\mathbf{C})^T \mathbf{Q} (\mathbf{CA} - \mathbf{A}_m\mathbf{C}) \\ \mathbf{W}_m &= (\mathbf{CA} - \mathbf{A}_m\mathbf{C})^T \mathbf{QCB} \\ \mathbf{R}_m &= \mathbf{B}^T\mathbf{C}^T\mathbf{QCB} + \mathbf{R} \end{aligned} \tag{15}$$

Therefore, by appropriate choice of the performance index matrices it is possible to guarantee implicit model following and thus desirable closed-loop behavior. The equivalent discrete-time version of the performance index Eq. (14) is an infinite sum over the sampled states and inputs (Bryson, 1994)

$$J = \sum_{j=0}^{\infty} \left(\tilde{\mathbf{x}}_j^T \mathbf{Q}_d \tilde{\mathbf{x}}_j + 2\tilde{\mathbf{x}}_j^T \mathbf{W}_d \tilde{\mathbf{u}}_j + \tilde{\mathbf{u}}_j^T \mathbf{R}_d \tilde{\mathbf{u}}_j \right) \tag{16}$$

The discrete versions of the weighting matrices, given the sampling time T_s, are calculated by (McLoan, 1978)

$$\begin{aligned} \mathbf{Q}_d &= \mathbf{Q}_m T_s + \left(\mathbf{Q}_m \mathbf{A} + \mathbf{A}^T \mathbf{Q}_m \right) T_s^2/2 + \cdots \\ \mathbf{W}_d &= \mathbf{W}_m T_s + \left(\mathbf{A}^T \mathbf{W}_m + \mathbf{Q}_m \mathbf{B} \right) T_s^2/2 + \cdots \\ \mathbf{R}_d &= \mathbf{R}_m T_s + \left(\mathbf{W}_m^T \mathbf{B} + \mathbf{B}^T \mathbf{W}_m \right) T_s^2/2 + \cdots \end{aligned} \tag{17}$$

where only the first two terms will be used to determine $\mathbf{Q}_d, \mathbf{W}_d, \mathbf{R}_d$ approximately.

To obtain a reconfigurable flight controller, it is desirable to design, independently, an unconstrained controller and a supervisory module that gives control corrections in case of an actuator fault. The key to this approach is the control law given by Eq. (4) (Rossiter *et al.*, 1998)

$$\tilde{\mathbf{u}}_j = \begin{cases} -\mathbf{K}_d \tilde{\mathbf{x}}_j + \mathbf{c}_j, & j = 0, \ldots, N-1 \\ -\mathbf{K}_d \tilde{\mathbf{x}}_j, & j = N, \ldots, \infty \end{cases} \tag{18}$$

Hence, optimization over $\{\tilde{\mathbf{u}}_j\}_{j=0}^{N-1}$ can be replaced by optimization over the stacked correction vector $\{\mathbf{c}_j\}_{j=0}^{N-1}$. Moreover, after N sampling times, it is desirable to have the state vector inside a terminal invariant set for the closed-loop system. This terminal constraint will guarantee the stability of the nominal closed-loop system (Mayne *et al.*, 2000). Therefore, considering the translated sets $\tilde{\mathbf{X}}$, $\tilde{\mathbf{U}}$ and $\tilde{O}_\infty$, the constrained optimal control problem to be solved is given by

$$\begin{aligned} &\min_{\{\mathbf{c}_j\}_{j=0}^{N-1}} J(\tilde{\mathbf{x}}_0) = \tilde{\mathbf{x}}_N^T \mathbf{P} \tilde{\mathbf{x}}_N + \sum_{j=0}^{N-1} \left(\tilde{\mathbf{x}}_j^T \mathbf{Q}_d \tilde{\mathbf{x}}_j + 2\tilde{\mathbf{x}}_j^T \mathbf{W}_d \tilde{\mathbf{u}}_j + \tilde{\mathbf{u}}_j^T \mathbf{R}_d \tilde{\mathbf{u}}_j \right) \\ &\quad \text{subject to:} \\ &\qquad \tilde{\mathbf{x}}_{j+1} = (\mathbf{\Phi} - \mathbf{\Gamma}\mathbf{K}_d)\tilde{\mathbf{x}}_j + \mathbf{\Gamma}\mathbf{c}_j, \qquad \tilde{\mathbf{x}}_N \in \tilde{O}_\infty \\ &\qquad \tilde{\mathbf{u}}_j = -\mathbf{K}_d \tilde{\mathbf{x}}_j + \mathbf{c}_j, \qquad j = 0, \ldots, N-1 \\ &\qquad \{\tilde{\mathbf{u}}_j\}_{j=0}^{N-1} \in \tilde{\mathbf{U}} \\ &\qquad \{\tilde{\mathbf{x}}_j\}_{j=0}^{N-1} \in \tilde{\mathbf{X}} \end{aligned} \tag{19}$$

where $\tilde{\mathbf{x}}_N^T \mathbf{P} \tilde{\mathbf{x}}_N = \sum_{j=N}^{\infty} \left(\tilde{\mathbf{x}}_j^T \mathbf{Q}_d \tilde{\mathbf{x}}_j + 2\tilde{\mathbf{x}}_j^T \mathbf{W}_d \tilde{\mathbf{u}}_j + \tilde{\mathbf{u}}_j^T \mathbf{R}_d \tilde{\mathbf{u}}_j \right)$ is the cost-to-go function and $\mathbf{P}$ is the terminal weight given by the solution of the discrete-time Riccati equation

$$\mathbf{P} = \mathbf{Q}_d + \mathbf{\Phi}^T \mathbf{P} \mathbf{\Phi} - \left(\mathbf{\Phi}^T \mathbf{P} \mathbf{\Gamma} + \mathbf{W}_d \right) \left(\mathbf{\Gamma}^T \mathbf{P} \mathbf{\Gamma} + \mathbf{R}_d \right)^{-1} \left(\mathbf{\Gamma}^T \mathbf{P} \mathbf{\Phi} + \mathbf{W}_d^T \right) \tag{20}$$

The feedback gain $\mathbf{K}_d$ is given by

$$\mathbf{K}_d = \left(\mathbf{\Gamma}^T \mathbf{P} \mathbf{\Gamma} + \mathbf{R}_d \right)^{-1} \left(\mathbf{\Gamma}^T \mathbf{P} \Phi + \mathbf{W}_d^T \right) \tag{21}$$

The solution of the problem expressed by Eq. (19) is possible predicting the state vector from the current discrete time to N moves ahead. These prediction equations, the constraints formulation and the conversion of the cost function into a quadratic programming problem can be found in the literature on MPC (Rossiter, 2003).

2.3 Observer for offset-free tracking

In the previous sections perfect knowledge of the state and disturbance vectors was assumed. Because they are not directly measured, an observer must be designed and employed. Consider the augmented plant model obtained from rewriting Eq. (1) as

$$\begin{aligned} \mathbf{x}_{a_{k+1}} &= \mathbf{\Phi}_a \mathbf{x}_{a_k} + \mathbf{\Gamma}_a \mathbf{u}_k \\ \mathbf{y}_k &= \mathbf{E}_a \mathbf{x}_{a_k} \end{aligned} \tag{22}$$

where $\mathbf{x}_{a_k} = \begin{bmatrix} \mathbf{x}_k^T & \mathbf{d}_k^T \end{bmatrix}^T$ and

$$\mathbf{\Phi}_a = \begin{bmatrix} \mathbf{\Phi} & \mathbf{\Gamma}_d \\ \mathbf{0} & \mathbf{I} \end{bmatrix}, \qquad \mathbf{\Gamma}_a = \begin{bmatrix} \mathbf{\Gamma} \\ \mathbf{0} \end{bmatrix}, \qquad \mathbf{E}_a = \begin{bmatrix} \mathbf{E} & \mathbf{0} \end{bmatrix} \tag{23}$$

The proposed linear state/disturbance observer has the form

$$\hat{\mathbf{x}}_{a_{k+1}} = \mathbf{\Phi}_a \hat{\mathbf{x}}_{a_k} + \mathbf{\Gamma}_a \mathbf{u}_k + \mathbf{L}\left(\mathbf{y}_k - \mathbf{E}_a \hat{\mathbf{x}}_{a_k}\right) \tag{24}$$

where the estimator gain matrix $\mathbf{L}$ is

$$\mathbf{L} = \begin{bmatrix} \mathbf{L}_x \\ \mathbf{L}_d \end{bmatrix} \tag{25}$$

Maeder and Morari (2007) propose the following algorithm to construct an observer $\mathbf{L}$.

1. Compute $\mathbf{L}_x$ such that $\mathbf{\Phi} - \mathbf{L}_x\mathbf{E}$ is stable and the pair $\left(\bar{\mathbf{H}}\mathbf{E}_a, \bar{\mathbf{\Phi}}\right)$ is detectable, where

$$\begin{aligned} \bar{\mathbf{H}} &= \mathbf{H}\left[\mathbf{E}\left(\mathbf{I} - \mathbf{\Phi} + \mathbf{\Gamma}\mathbf{K}_d\right)^{-1}\mathbf{L}_x + \mathbf{I}\right] \\ \bar{\mathbf{\Phi}} &= \mathbf{\Phi}_a - \begin{bmatrix} \mathbf{L}_x^T & \mathbf{0} \end{bmatrix}^T \mathbf{E}_a \end{aligned} \tag{26}$$

2. Compute $\bar{\mathbf{L}}_d$ such that $\mathbf{I} - \bar{\mathbf{L}}_d\bar{\mathbf{H}}\mathbf{E}\left(\mathbf{I} - \mathbf{\Phi} + \mathbf{L}_x\mathbf{E}\right)^{-1}\mathbf{\Gamma}_d$ is stable

3. Compute the following matrix

$$\mathbf{T} = \begin{bmatrix} \mathbf{I} & -\left(\mathbf{I} - \mathbf{\Phi} + \mathbf{L}_x\mathbf{E}\right)^{-1}\mathbf{\Gamma}_d \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \tag{27}$$

4. Calculate the estimator gain $\mathbf{L}$ with

$$\mathbf{L} = \begin{bmatrix} \mathbf{L}_x \\ \mathbf{0} \end{bmatrix} + \mathbf{T}^{-1}\begin{bmatrix} \mathbf{0} \\ \bar{\mathbf{L}}_d\bar{\mathbf{H}} \end{bmatrix} \tag{28}$$

The proposed algorithm allows to design $\mathbf{L}_x$ first and then the disturbance observer. Hence, the eigenvalues of $\mathbf{\Phi} - \mathbf{L}_x\mathbf{E}$ (i.e. the state observer dynamics) are not affected by the disturbance observer dynamics. Also, this observer design allows offset-free control despite the effects of disturbances and model uncertainty, assuming that no constraints are active at steady state. As explained previously, this condition might not be observed when one or more actuators are jammed at a fixed position. Then, the elimination of the offset error will strongly depend on the analytical redundancy (Shead *et al.*, 2008).

3. TRAJECTORY-TRACKING FAULT-TOLERANT FLIGHT CONTROL

The trajectory-tracking system implemented in the ATTAS ground simulator is presented in Fig. 2. The desired path to be flown is defined by a sequence of waypoints in geodetic coordinates. The trajectory generator produces a time-parametrized smoothed trajectory in cartesian coordinates (Almeida, 2008). Then, based on the present position of the aircraft, vertical and lateral guidance are computed and sent to the autopilots.

The longitudinal autopilot uses the calibrated airspeed command and vertical guidance to manipulate the sum of left and right power lever angles $pla_t = pla_l + pla_r$, and the elevator command δ_{e_c}. In the lateral autopilot the feasible target-tracking model predictive control (FTT-MPC) technique was applied . The autopilot manipulates the aileron command δ_{a_c}, rudder command δ_{r_c} and the difference of left and right power lever angles $pla_a = pla_l - pla_r$ in order to track yaw rate $\dot{\psi}_c$ and lateral load factor n_y.

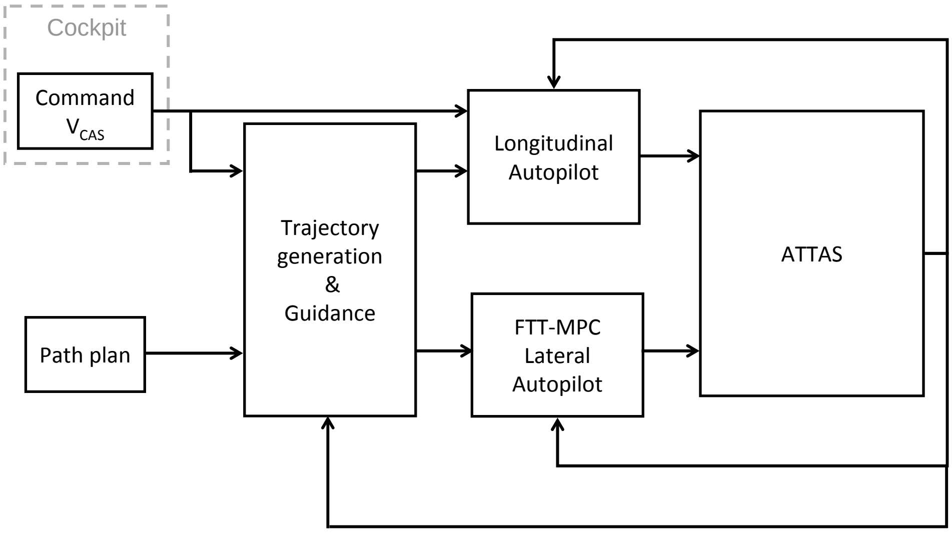


Figure 2. Trajectory-tracking fault-tolerant flight control

3.1 Controller design

The longitudinal autopilot was designed using single-input, single-output (SISO) techniques (Stevens and Lewis, 2003). The fault-tolerant lateral autopilot was constructed considering an ATTAS linear model obtained with calibrated airspeed V_{CAS} =145 kt, pressure altitude of 1,000 m, gear up and flaps 1 deg. The dynamics of the effectors and their limitations are taken into consideration. Thus, the state vector is

$$\mathbf{x}_{lat} = \begin{bmatrix} p & r & \phi & v_b & \delta_a & \delta_r & n_{1_a} \end{bmatrix}^T \tag{29}$$

where p is the roll rate (rad/s), $r \cong \dot{\psi}$ is the yaw rate (rad/s), ϕ is the bank angle (rad), v_b is the lateral velocity component in the body-fixed y-axis (m/s), δ_a is the aileron deflection (rad), δ_r is the rudder deflection (rad), and $n_{1_a} = n_{1_l} - n_{1_r}$ is the asymmetrical fan speed, i.e. the difference of left and right fan speeds. The control vector is $\mathbf{u}_{lat} = \begin{bmatrix} \delta_{a_c} & \delta_{r_c} & pla_a \end{bmatrix}^T$, which concatenates the aileron command (rad), rudder command (rad), and asymmetrical power lever angle (rad).

The performance output defined by Eq. (10) is $\mathbf{z}_m = \begin{bmatrix} p & r & \phi & v_b \end{bmatrix}^T$, that is required to follow the transient behavior specified by the model

$$\mathbf{A}_m = \begin{bmatrix} -2.24 & 1.49 & -0.868 & -0.0364 \\ -0.169 & -0.765 & -0.0253 & 0.0257 \\ 1.00 & 0.102 & 0 & 0 \\ 8.11 & -77.4 & 9.60 & -0.186 \end{bmatrix} \tag{30}$$

where $\mathbf{A}_m$ has eigenvalues $\{-1.7 \quad -0.553 \pm 1.55i \quad -0.386\}$. It should be noted that no actuator dynamical mode is required to follow a model, only the rigid-body aircraft lateral modes. The matrices $\mathbf{Q}$ and $\mathbf{R}$ of Eq. (15) were set as $\mathbf{Q} = 1 \times 10^5 \mathbf{I}$ and

$$\mathbf{R} = \begin{bmatrix} 1 \times 10^{-3} & 0 & 0 \\ 0 & 1 \times 10^{-3} & 0 \\ 0 & 0 & 10 \end{bmatrix} \tag{31}$$

where the third diagonal term, related to the power lever command, was selected to give almost no usage of asymmetrical thrust in the nominal (without faults) condition, but to provide proper power lever command allocation in case of fault of aileron and/or rudder. The sampling period chosen was $T_s = 0.035$ s. After discretization of the dynamical system and the weighting matrices, the following feedback gain was obtained

$$\mathbf{K}_d = \begin{bmatrix} -0.42 & 0.013 & -0.77 & -0.0011 & 1.81 & -0.0062 & -0.0019 \\ 0.27 & -1.8 & 0.18 & 0.013 & -0.0038 & 1.7 & -0.002 \\ -0.0036 & -0.031 & -0.0088 & -0.0002 & 0.049 & 0.079 & 0.14 \end{bmatrix} \tag{32}$$

The target calculation problem assumes that the controlled variables are $\mathbf{z} = \begin{bmatrix} \dot{\psi} & n_y \end{bmatrix}^T$ and also that the full state is observed, which leads, with respect to Eq. (1), to $\mathbf{E} = \mathbf{I}$ and

$$\mathbf{H} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0.0179 & 0.104 & 0 & -0.0176 & 0 & 0.259 & 0 \end{bmatrix} \tag{33}$$

The weighting matrices of Eq. (3) were chosen as $\mathbf{Q}_{ss} = \mathbf{I}$ and $\mathbf{R}_{ss} = 1 \times 10^{-10}\mathbf{I}$. The disturbance model was chosen with two variables and the related distribution matrix $\mathbf{\Gamma}_d$ was built taking the first and the third columns of $\mathbf{\Gamma}$. This (arbitrary) choice guarantees that the augmented system given by Eq. (23) is detectable. The observer gain $\mathbf{L}$ was obtained from the proposed algorithm, where both $\mathbf{L}_x$ and $\bar{\mathbf{L}}_d$ were determined via LQR formulations.

The admissible sets related to the fault-tolerant control problem must be established. In the nominal conditions, the set of admissible controls is defined on the basis of requirements and aforementioned reasons

$$\begin{aligned} -40^\circ \leq & pla_a \leq 40^\circ \\ -15\%\, T_s \leq & \Delta n_{1_{l,r}} \leq 15\%\, T_s \\ -15^\circ \leq & \delta_{a_c} \leq 15^\circ \\ -10^\circ \leq & \delta_{r_c} \leq 10^\circ \end{aligned} \tag{34}$$

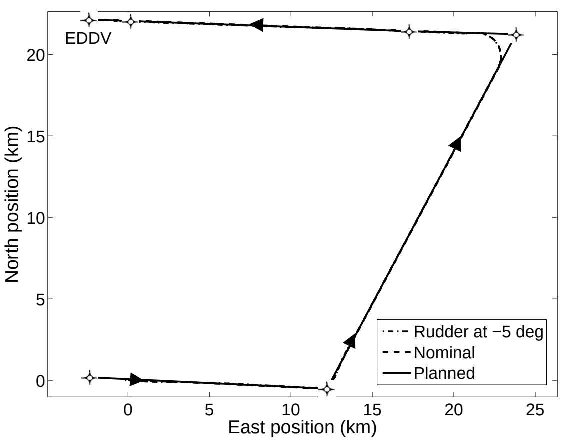
Each fault condition is reflected into new constraints of the target calculator and the constrained regulator. The computation of the related feasible target and terminal invariant sets is performed off-line. In this work, as failure case the rudder jammed at -5° has been simulated. Therefore, the set $\mathbf{X}_N$ is calculated for both nominal and failed condition through available numerical techniques (Kvasnica *et al.*, 2004). The slice of $\mathbf{X}_N$ to compute the set $\mathbf{X}_{ss_N}$ is done on-line. A similar procedure is adopted for the terminal invariant set of the constrained regulator (Almeida and Leissling, 2009).

The choice of the control horizon is driven by the computational capability of the flight control hardware. The proposed technique guarantees feasibility of the constrained regulator problem, thus the value of N only affects how the demand is filtered by the target calculator. For both nominal and fault conditions, a control horizon $N = 2$ has been defined.

4. GROUND SIMULATOR IMPLEMENTATION AND SIMULATION RESULTS

The proposed system was implemented in Simulink® environment. The MPC constrained optimization problem was solved through a quadratic programming solver. The active set method of Goldfarb and Idnani (Fletcher, 1987) was converted into a S-function, which could be compiled for several target environments. After that, an executable file has been built, using Real-Time Workshop®, which could be tested using the ATTAS ground-based simulator. A detailed description of this procedure and the interfaces of the experimental computer with the flight control system of the ground simulator is given by Gestwa *et al.* (2003).

The ILS approach of runway 26L at Hannover airport (EDDV) was chosen to be simulated. The calibrated airspeed was maintained constant at 145 KCAS. The circular path to smooth the transitions of waypoints is an inertially referenced path with 3 deg/s of nominal turn rate. Fig. 3 shows the horizontal and vertical profiles obtained from simulation in both cases (nominal and rudder failure), where the initial position of the aircraft has cartesian coordinates (0,0). There is no significant difference between both trajectories.



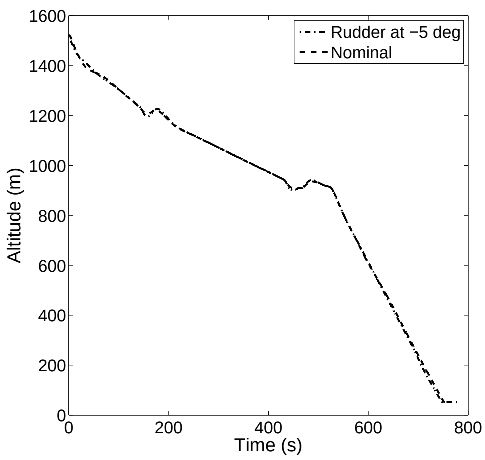


Figure 3. Horizontal and vertical views of the planned and simulated path

Figure 4 shows adequate performance of the lateral autopilot in all situations. The yaw rate demanded by the guidance system, and zero lateral acceleration, were correctly followed by the fault-tolerant autopilot, even with a considerable range of operation of altitude. Figure 5 shows the corrections computed by the feasible target calculator with rudder fault, generating admissible target state and control vectors. It is important to emphasize that, without such corrections, the constrained regulation problem would be unfeasible.

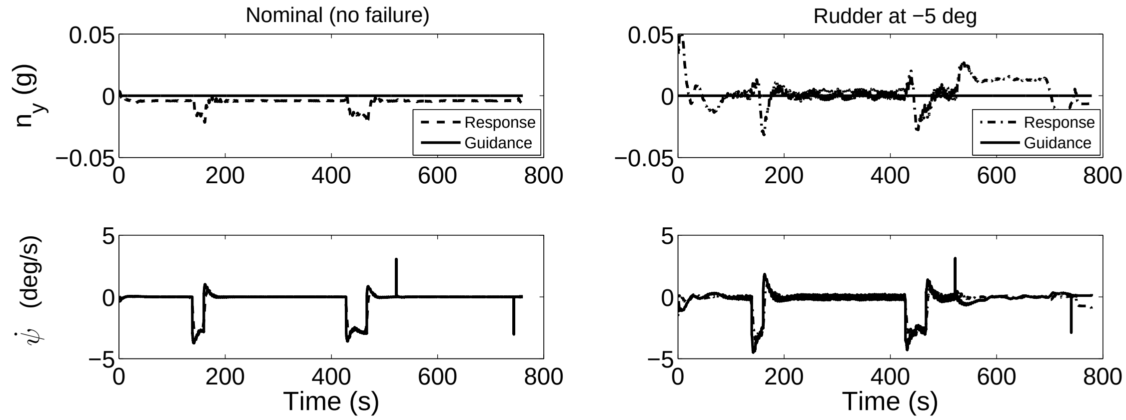


Figure 4. Guidance commands and responses

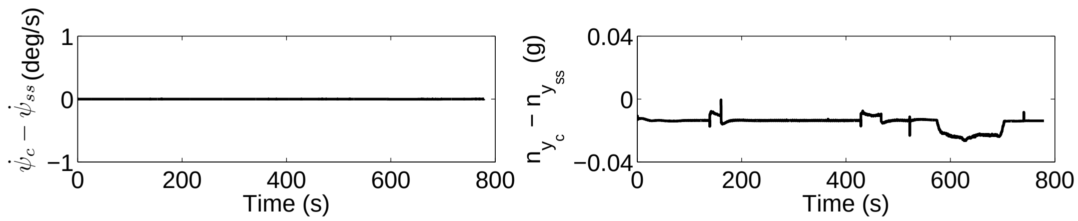


Figure 5. Corrections computed by the feasible target calculator in the fault scenario

The dynamical control allocation performed by the lateral autopilot is depicted in Fig. 6. Oscillations in the asymmetrical fan speed occurred because of nonlinearities of the engine model. Nevertheless, the desired reconfigurability is achieved through usage of asymmetrical thrust only in the condition of jammed rudder.

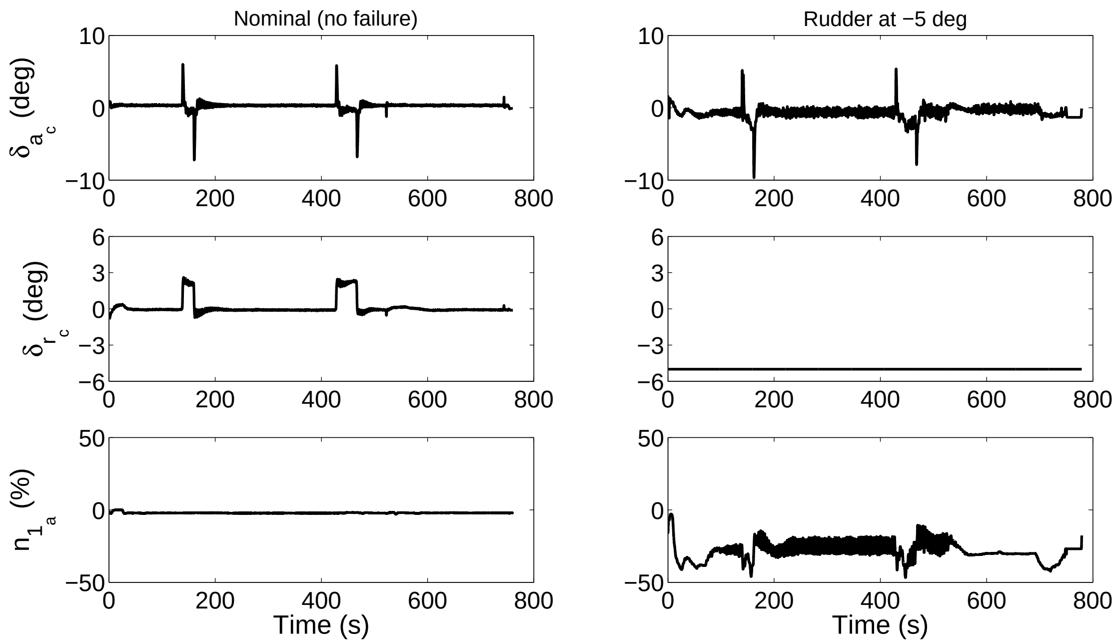


Figure 6. Aileron / rudder commands and asymmetrical fan speed

5. CONCLUSIONS

In this paper, a novel fault-tolerant model predictive controller is proposed. This controller ensures feasibility by means of filtering the desired reference during the constrained target calculation. Thus, the usual way to enlarge the domain of attraction of a MPC by increasing the control horizon is avoided, making the proposed controller adequate for flight control systems with limited computational capability. Also, the constrained regulator of the controller is designed to minimize a cost function which penalizes the deviation from a desired reference dynamical model. Simulation results obtained from the ATTAS ground simulator showed proper control redistribution and reconfiguration, making use of asymmetrical thrust only after the occurrence of a rudder fault.

6. ACKNOWLEDGEMENTS

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8. RESPONSIBILITY NOTICE