RESULTS OF LATERAL-DIRECTIONAL HELICOPTER SYSTEM IDENTIFICATION USING OUTPUT-ERROR AND BOTH GENETIC AND LEVENBERG-MARQUARDT OPTIMIZATION ALGORITHMS

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Abstract. This article presents the results concerning the estimation of the uncoupled linear lateral-directional forward flight derivatives of the "Twin Squirrel" AS355-F2 helicopter. The identification problem is solved in the time domain using the output error approach, in conjunction with both genetic and Levenberg-Marquardt optimization algorithms. A flight test compaing was performed and experimental data was obtained for this analysis. The helicopter linear equations of lateral-directional motion are used in order to identify the dimensional derivatives in level flight at 5,000 ft and 80 kt. The maneuvers were specified in accordance with conventional flight test procedures, taking in account the limitations of the mathematical model and flight safety constraints.

Keywords: Helicopter System Identification, Lateral-Directional Mode, Output Error, Genetic Optimization Algorithm, Levenberg-Marquardt Optimization Algorithm

1. INTRODUCTION

System identification techniques applied to stability and control derivatives estimation of fixed and rotary wing aircrafts are well developed and commonly used in many research centers and aeronautical industries around the world, as shown, for instance, by Jategaonkar *et al.* (2004) and Wang and Iliff (2006). More specifically for helicopters, Hamel and Kaletka (1997) present a general vision of this aircraft system identification progress up to 1997 and Padfield (1996) describes a comprehensive flight dynamic theoretical model, flight qualities criteria development, flight test techniques and several results of his research in the United Kingdom.

More recently, Tischler and Remple (2006) also describe the helicopter dynamic systems identification state-of-theart in their book "Aircraft and Rotorcraft System Identification". In this reference work emphasis is given on engineering methods and interpretations of flight test results using a system identification software package in the frequency-domain, known as CIFER@ (Comprehensive Identification from Frequency Responses). In the frequency response methodology, accurate mathematical modeling of the aircraft starts with flight test data collected in the time domain. The reliability of the identified model is also proved by a time-domain verification method using different flight test data.

All above related works use local optimization algorithms based on gradient method as Gauss-Newton and Levenberg-Marquadt search methods for finding a minimum of a function. Concerning global optimization algorithms and more specifically, genetic algorithms, they are used by Hajela and Lee (1995) in rotor blade design and by Wells *et al.* (1995) in the acoustic level reduction rotor design. Regarding helicopter system identification techniques, a few works utilize genetic algorithms in search and optimization. In this area, it can be cited Cruz *et al.* (2006, 2007) in the longitudinal mode system identification of the AS355-F2 helicopter (Twin Squirrel) and del Cerro *et al.* (2005) in the identification of a small unmanned helicopter model.

This work deals with lateral-directional system identification of the AS355-F2 helicopter (Twin Squirrel), using the time-domain output-error approach combined with both genetic and Levenberg-Marquardt algorithm optimization. The rotorcraft identification methodology used in this work is the well-known *Quad*-M or M^4V methodology, proposed by Hamel and Jategaonkar (1996) and shown schematically in Fig. 1. This methodology takes in account the main elements of rotorcraft system identification, including the rotorcraft excitation maneuvers, the data measurements, the mathematical models from the helicopter equations of motion and the parameter estimation methods through output-error minimization. Finally, the identified rotorcraft model validation is done from the complimentary flight test data. Each one of these elements will be discussed on the following sections in order to clarify the utilized system identification methodology.



Figure 1. M⁴V output error system identification methodology - Hamel and Jategaonkar (1996)

2. MANEUVERS

The choice of the proper flight test maneuvers, by shaping the excitation signals, is very important to minimize the uncertainties in the parameter estimation procedures and to maximize the flight test data content. The optimization of the excitation signal can be realized from a priori knowledge of the initial dynamic parameters of interest. However, since there are no priori studies available for AS355-F2 helicopter, the maneuvers were specified applying conventional flight test procedures and taking into account flight safety constraints. Since this work focuses the determination of the lateral-directional flight derivatives at forward and level flight, special sequences of sharp-edged pulses known as the "3-2-1-1" using lateral cyclic and pedal inputs were used to excite the dutch-roll mode with 80 kt indicated airspeed and 5,000 ft of pressure altitude.

The identification procedure used a "3-2-1-1" sequence with both lateral cyclic and pedal inputs, while the validation procedure utilized two different sequences (one with lateral cyclic inputs and pedal fixed and vice versa).

3. FLIGHT TEST DATA MEASUREMENTS

The tested helicopter was equipped with an Aydin Vector Data Acquisition System (AVDAS), the ATD-800 digital recorder and a flight-test air data system, mounted on a nose boom, as depicted in Fig. 2. This system measures thirty-five different parameters with a sampling rate from 18 to 72 Hz. Some of the data measurements are: fuel quantity from both tanks, nose boom static and dynamic pressures, external temperature, aerodynamic angle of attack and sideslip, roll, pitch and yaw rates (p, q and r, respectively), load factors, longitudinal and lateral attitudes (θ and ϕ), heading (ψ), collective, longitudinal and lateral cyclic and pedal input deflections (δ_c , δ_B , δ_A and δ_P , respectively).



Figure 2. "Squirrel" AS355-F2 helicopter and installed nose boom

Finally, the Earth axis speeds (u_0 , v_0 and w_0) are gotten by means of the Z12 Differential Global Positioning System (DGPS), from Astech, whose antenna is fixed in the vertical fin upper. The DGPS and AVDAS data synchronization is made by means of an event simultaneous register. Still, the DGPS data is represented with the same AVDAS data cadence by means of the linear interpolation. The wind direction and intensity are obtained comparing the body axis

speeds with the aerodynamic speed from the flight-test air data system, mounted on a nose boom, at trim conditions. Consequently, the body axis speeds (u, v and w) is easily calculated adding wind vector to the Earth axis speeds.

4. HELICOPTER LATERAL-DIRECTIONAL MODEL

The helicopter equations of motion, deduced from Newton second law for translational and rotational movements, are given by Prouty (1989) and Cooke and Fitzpatrick (2002).

This work deals only with the stability and control derivatives estimation of the lateral-directional mode. Then, the uncoupled dynamic equations are given as:

$$Y = -mg\cos\theta\sin\phi + m(\dot{v} - pw + ru) \tag{1}$$

$$L = I_{xx}\dot{p} - I_{zx}(\dot{r} + pq) - (I_{yy} - I_{zz})qr$$
(2)

$$N = I_{zz} \dot{r} - I_{zx} (\dot{p} - qr) - (I_{xx} - I_{yy}) pq$$
(3)

where *Y* represents the lateral external force, *L* and *N* are, respectively, the roll and yaw moments and $I_{()}$ corresponds to the moments and product of inertia of a rotating body. The kinematic relation for the roll rate about the Y-axis is written as:

$$\phi = p + q \sin \phi tg\theta + r \cos \phi tg\theta \tag{4}$$

These equations are clearly non-linear, but a meaningful analysis can be realized by converting into linear differential equations, considering only the small-perturbations around trimmed equilibrium points (represented by subscript 0) in the rotorcraft flight envelop. In matrix notation, the linearized lateral-directional model is given by:

$$\begin{bmatrix} \Delta \dot{\nu} \\ \Delta \dot{p} \\ \Delta \dot{\phi} \\ \Delta \dot{r} \end{bmatrix} = \begin{bmatrix} \frac{Y_{\nu}}{m} & \frac{Y_{p}}{m} + w_{0} & g \cos\phi_{0} \cos\theta_{0} & \frac{Y_{r}}{m} - u_{0} \\ L_{\nu}' & L_{p}' & 0 & L_{r}' \\ 0 & 1 & 0 & \cos\phi_{0} t g \theta_{0} \\ N_{\nu}' & N_{p}' & 0 & N_{r}' \end{bmatrix} \begin{bmatrix} \Delta \nu \\ \Delta \phi \\ \Delta r \end{bmatrix} + \begin{bmatrix} \frac{Y_{\delta_{A}}}{m} & \frac{Y_{\delta_{p}}}{m} \\ L_{\delta_{A}} & L_{\delta_{p}}' \\ 0 & 0 \\ N_{\delta_{A}}' & N_{\delta_{p}}' \end{bmatrix} \begin{bmatrix} \Delta \delta_{A} \\ \Delta \delta_{P} \end{bmatrix}$$
(5)

where Δ represents the variation from initial trim condition, $Y_{()}$ is the dimensional lateral force stability and control derivatives, and $L'_{()}$ and $N'_{()}$ corresponds to the dimensional roll and yaw moment stability and control primed derivatives, respectively.

It should be observed that Eq. (5) presents lateral cyclic and pedal input deflections instead of the main rotor lateral pitch and tail rotor collective angles, respectively, because the tested helicopter instrumentation doesn't measure these parameters. The AS355-F2 control system is hydraulically actuated and its transfer function is modeled only as a time delay. Therefore, Eq. (5) may also be written as:

$$\dot{x} = Ax + B\overline{u}(t-\tau) + \dot{x}_{bias}$$
(6)

where \dot{x}_{bias} is the constant and unknown bias vector that was included in the state-space equation in order to provide a first-order correction for the random errors due non-measurements inputs such as turbulences and noise, and τ is the time-delay associated with unmodeled dynamics, such as actuator dynamics, control linkages and transient rotor dynamics.

The observation vector (y) is given by Eq. (7), where x is the state vector and y_{ref} is a constant vector included in order to compute offsets in steady state due sensor misalignments and errors and several other influences:

$$y = x + y_{raf} \tag{7}$$

Therefore, taking into account y_{ref} , there are twenty-five parameters to be identified as shown by the following vector of unknown parameters:

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$$\Theta_{ld} = \begin{bmatrix} \underline{Y}_{v}, \underline{Y}_{p}, \underline{Y}_{r}, L'_{v}, L'_{p}, L'_{r}, N'_{v}, N'_{p}, N'_{r}, \frac{Y_{\delta_{A}}}{m}, \frac{Y_{\delta_{P}}}{m}, L'_{\delta_{A}}, L'_{\delta_{P}}, N'_{\delta_{A}}, N'_{\delta_{P}}, \\ \Delta \dot{v}_{bias}, \Delta \dot{p}_{bias}, \Delta \dot{\phi}_{bias}, \Delta v_{ref}, \Delta p_{ref}, \Delta \phi_{ref}, \Delta r_{ref}, \tau_{A}, \tau_{P} \end{bmatrix}$$

$$(8)$$

5. PARAMETER ESTIMATION METHOD

5.1. Output error approach

The output error approach was used to estimate the vector of unknown parameters presented in Eq. (8). As shown in Fig. 1, the basic principle of this methodology is to minimize the error between the in-flight measured responses and the estimated results of the identified mathematical model submitted to the observed inputs. This minimization process is made as function of the dynamic model parameters, such as aerodynamic derivatives, sensor bias and sensitivities.

Therefore, a cost function is defined which measures the matching of the real and simulated data, when a certain group of adjustable parameters (e.g. the coefficients of the proposed dynamic model) are varied. An iterative search method is employed to vary this group of parameters such as to minimize the adopted cost function, usually taken as a quadratic function of the predicted matching error.

Let f be the cost function, and consider Θ_{ld} as being the vector of parameters to be estimated, \overline{u} the system input vector, y the observation vector, and let $y_{sim}(x_i; \Theta_{ld 1}..., \Theta_{ld M})$, where i = 1,..., N, be the output of the proposed model for a determined group of parameters Θ_{ld} . Then, the problem becomes how to determine the vector of adjustable parameters Θ_{ld} that minimizes the cost function given by $f = f(y_{sim}(x_i; \Theta_{ld 1}..., \Theta_{ld M}), y)$.

There are many methods to calculate the cost function, including the maximum likelihood (MLE) and the nonlinear least-squares. The least-squares optimization procedure is basically a minimum square error adjustment given by:

$$f = \sum_{i=1}^{N} \left[y_i - y_{sim} \left(x_i; \vec{\Theta}_{id} \right) \right] \left[y_i - y_{sim} \left(x_i; \vec{\Theta}_{id} \right) \right]^T$$
(9)

On the other hand, the MLE method takes into account the measurement noise variance to weight the system outputs during the optimization procedure through the measurements noise covariance matrix, FF. Then, suppose that each output value, y_i , has an associated random measurement error with a normal distribution around the "true" value y_{sim} , the conditional probability density function $p((y_i - y_{sim})/\Theta_{ld})$ is written by:

$$p((\vec{y} - \vec{y}_{sim}) / \vec{\Theta}_{ld}) = \frac{1}{(2\pi)^{M/2} / FF^T} e^{N/2} \cdot exp\left\{-\frac{1}{2} \sum_{i=1}^{N} \left[y_i - y_{sim}(x_i; \vec{\Theta}_{ld})\right]^T \left[FF^T\right]^{-1} \left[y_i - y_{sim}(x_i; \vec{\Theta}_{ld})\right]\right\}$$
(10)

where FF is the covariance matrix given for:

$$FF^{T} = \frac{1}{N} \sum_{i=1}^{N} \left[y_{i} - y_{sim} \left(x_{i}, \vec{\Theta}_{ld} \right) \right] \left[y_{i} - y_{sim} \left(x_{i}, \vec{\Theta}_{ld} \right) \right]^{T}$$
(11)

From the conditional probability density function equation, it can be obtained a modified cost function, applying the negative of the natural log on both sides:

$$f(\vec{\Theta}) = \frac{1}{2} \left\{ \sum_{i=1}^{N} \left[y_i - y_{sim} \left(x_i, \vec{\Theta}_{ld} \right) \right]^T \left[FF^T \right]^{-1} \left[y_i - y_{sim} \left(x_i, \vec{\Theta}_{ld} \right) \right] + \ln \left| FF^T \right| \right\}$$
(12)

Equation (12) shows that the parameter identification problem becomes a quadratic function minimization problem, since the maximization of the likelihood function is equivalent to minimize a weighted least square equation. Taking into account that the covariance matrix is constant, the cost function can be simplified to the form:

$$f(\vec{\Theta}) = \frac{1}{2} \left\{ \sum_{i=1}^{N} \left[y_i - y_{sim} \left(x_i, \vec{\Theta}_{ld} \right) \right]^T \left[FF^T \right]^{-1} \left[y_i - y_{sim} \left(x_i, \vec{\Theta}_{ld} \right) \right] \right\}$$
(13)

The determination of a parameter vector Θ_{ld} that minimize the cost function given by Eq. (13) is equivalent to find a parameter vector that maximize the probability of measuring, $y_i = (y_i)_{simb}$ i = 1, ..., N. Comparing Eq. (9) and (13), it can be concluded that the least squares cost function, except for the multiplication factor, is given by Eq. (13), replacing FF

by the identity matrix. Therefore, in order to implement the weighted least-square cost function, it is enough to replace FF by a diagonal matrix to consider different weights to each state. In this work, it was developed a Matlab[®] program to minimize the cost function defined by both weighted least square and MLE errors.

5.2. Genetic algorithm optimization

Classic methods to solve the maximum likelihood minimization problem are Levenberg-Marquadt, Gauss-Newton, Newton-Raphson, among others. This article proposes a new approach by using the Matlab[®] Genetic Algorithm (GA) and the Direct Search Toolbox in order to estimate the helicopter lateral-directional derivatives combined with the Levenberg-Marquardt optimization algorithm, as shown schematically in Fig. 3. In case of there is no priori information available, this methodology states that it should be applied initially the genetic algorithm and after that a classic method in order to solve Eq. (13). This is especially important since that the parameters estimation procedure becomes less sensitive to the initial values. Besides, another interesting characteristic is that the solution obtained by the genetic algorithm is globally optimal.



Figure 3. Combined genetic and Levenberg-Marquadt optimization algorithms approach

Based on biological evolution principles, the genetic algorithms are structured random search techniques for optimization. Therefore, they utilize concepts such as population size, individual characterization and processes related to selection, reproduction, crossover and mutation of the individuals.

The GA optimization process starts by creating a matrix, representing an initial population *IP*, with each line formed by randomly chosen individual with fixed-length character strings ($n_{\Theta td}$), as shown below,

| | θ^{1}_{ld1} | θ^1_{ld2} | θ^1_{ld3} | ••• | $\theta^1_{ld n_{\theta ld}}$ |
|------|--------------------------|--------------------------|--------------------------|-----|--------------------------------|
| | θ_{ld1}^2 | θ_{ld2}^2 | θ_{ld3}^2 | | $\theta^2_{ldn_{\theta ld}}$ |
| IP = | θ_{ld1}^3 | θ_{ld2}^3 | θ_{ld3}^3 | ÷ | $\theta^3_{ldn_{\Theta M}}$ |
| | 1 : | ÷ | ÷ | | : |
| | $\theta_{ld1}^{n_{ind}}$ | $\theta_{ld2}^{n_{ind}}$ | $\theta_{ld3}^{n_{ind}}$ | | $\theta_{ldn_{out}}^{n_{ind}}$ |

where $n_{\Theta b d}$ represents a twenty-five components vector as shown in Eq. (8). After simulations with different number of individuals (n_{ind}), this work uses two hundred individuals at each generation due to adequate computation time and convergence of the fitness function.

The following step is to assign a fitness value to each individual in the population using a fitness measure, represented by the cost function of Eq. (13). A new population is created by applying three genetic operations, depicted in Fig. 4, to each individual string of the current generation. The individuals generated by the genetic processes are:

- Elite: formed by individuals in the current generation those are copied to the new generation based on their fitness properties;

- Crossover: new individuals, created by genetically recombining two substrings, using the crossover operation at a randomly chosen crossover point. The newly created individual inherits the characteristics from the original chosen pair; and

- Mutation: new individuals are created by randomly changing a few strings chosen at random positions in the original string.



Figure 4. Reproduction Instruments

The GA iteratively performs the above steps until a termination criterion is satisfied. In this work, the maximum number of generations was used for this purpose.

Concerning the GA implementation in the Matlab[®] environment, each current generation is composed of 2 elite individuals, with the rest of the population formed by crossover and mutation processes in the proportion of 80% and 20%, respectively.

The functions that create the initial population and produce mutation children were developed from a uniform distribution of random numbers, generated on a specified interval: $\pm 100\%$ of the B0-105 derivatives presented by Heffley *et al.* (1979). The output function is evaluated over a subdomain of individuals that attends the limits associated with bounds and stability constraints, since the aerodynamic parameters should be on the cited interval and the dutchroll dynamics is known to be stable on this helicopter.

5.2. Levenberg-Marquardt optimization algorithm

After genetic search verification, this article also proposes the Levenberg-Marquardt optimization algorithm presented by Mulder *et al.* (1999) and Press *et al.* (1988). Therefore, the cost function given by Eq. (13) may be minimized iteratively according to:

$$\hat{\Theta}_{ld}^{(j)} = \hat{\Theta}_{ld}^{(j-1)} + \lambda^{(j-1)} \xi^{(j-1)}$$
(15)

where $\hat{\Theta}_{ld}^{(j)}$ represents the estimate vector at *j*th iteration, $\lambda^{(j-I)}$ is the scalar chosen in accordance with the algorithm proposed by Press *et al.* (1988) for a reduced value of $f(\vec{\Theta}_{ld})$ and $\xi^{(j-I)}$ is based on information about the cost function acquired at previous iterations. Numerically, $\xi^{(j-I)}$ is often obtained using the values of the first- and the second-order gradients of $f(\vec{\Theta}_{ld})$ with respect to parameters vector Θ_{ld} , according to:

$$\xi^{(j-1)} = -\left[f''\left(\vec{\Theta}_{ld}^{(j-1)}\right)\right]^{-1} f'\left(\vec{\Theta}_{ld}^{(j-1)}\right) = -\left[\frac{\partial^2 f\left(\vec{\Theta}_{ld}\right)}{\partial \vec{\Theta}_{ld}}\right]^{-1} \frac{\partial f\left(\vec{\Theta}_{ld}\right)}{\partial \vec{\Theta}_{ld}} \bigg|_{\vec{\Theta}_{ld} = \vec{\Theta}_{ld}^{(j-1)}}$$
(16)

A difficulty with the Eq. (16) is that Hessian matrix $f''(\bar{\Theta}_{ld})$ may not be positive definite and thus not point in a "downhill" direction. Mulder *et al.* (1999) propose to replace the $f''(\bar{\Theta}_{ld})$ by its expectation *R*, known as the Fisher information matrix that can be shown to be non-negative definite. Therefore, the Eq. (15) becomes:

$$\hat{\Theta}_{id}^{(j)} = \hat{\Theta}_{id}^{(j-1)} - \lambda^{(j-1)} R^{-1} \left(\hat{\Theta}_{id}^{(j-1)} \right) f' \left(\hat{\Theta}_{id}^{(j-1)} \right)$$
(17)

The first-order gradient of the likelihood function in Eq. (17) can be derived from Eq. (13) as:

$$f'(\hat{\Theta}_{ld}^{(j-1)}) = \sum_{i=1}^{N} \frac{\partial \left[y_i - y_{sim}(x_i, \vec{\Theta}_{ld}) \right]^T}{\partial \vec{\Theta}_{ld}} \bigg|_{\vec{\Theta}_{ld} = \hat{\Theta}_{ld}^{(j-1)}} \left[FF^T(\hat{\Theta}_{ld}^{(j-1)}) \right]^{-1} \left[y_i - y_{sim}(x_i, \hat{\Theta}_{ld}^{(j-1)}) \right]$$
(18)

Applying the following equivalency presented by Goodwin and Payne (1977)

$$E\left\{f''\left(\hat{\Theta}_{ld}^{(j-1)}\right)\right\} = E\left\{f'\left(\hat{\Theta}_{ld}^{(j-1)}\right)\left[f'\left(\hat{\Theta}_{ld}^{(j-1)}\right)\right]^{T}\right\}$$
(19)

the Fisher information matrix may be obtained as:

$$R\left(\hat{\Theta}_{ld}^{(j-1)}\right) = E\left\{f''\left(\hat{\Theta}_{ld}^{(j-1)}\right)\right\} = \sum_{i=1}^{N} \frac{\partial \left[y_i - y_{sim}\left(x_i, \vec{\Theta}_{ld}\right)\right]^T}{\partial \vec{\Theta}_{ld}} \bigg|_{\vec{\Theta}_{ld} = \hat{\Theta}_{ld}^{(j-1)}} \left[FF^T\left(\hat{\Theta}_{ld}^{(j-1)}\right)\right]^{-1} \frac{\partial \left[y_i - y_{sim}\left(x_i, \vec{\Theta}_{ld}\right)\right]}{\partial \vec{\Theta}_{ld}}\bigg|_{\vec{\Theta}_{ld} = \hat{\Theta}_{ld}^{(j-1)}}$$
(20)

The iteration starts with initial estimates of all unknown parameters and computes parameter updates according to Eq. (17) based on Eq. (18) and (20). Assuming initially a modest value for λ , as 0.001, on Eq. (17), the cost function is evaluate and if its value is greater than or equal to the last value, so λ is increased by a factor of 10 and the program returns to evaluate the cost function until to obtain a value less than the last value. Also necessary is a condition for stopping, so all these procedures repeat until that the cost function has been reached at least five significant digits equals to the last value.

6. PARAMETER ESTIMATION RESULTS

The 3-2-1-1 excitation maneuver using lateral cyclic and pedal inputs was implemented in the flight test, under a nominal flight operation point of 80 kt and 5,000 ft. Figure 5 shows the measured deflection inputs and the measured and simulated responses of lateral body axis speed v, roll rate p, lateral attitude ϕ and yaw rate r.



Figure 5. Results at 80 kt and 5,000 ft

The correlation coefficient between measured (y) and simulated data (y_{sim}), defined as the normalized crosscovariance function $\rho_{y_{v_{sim}}}$, as defined by Bendat and Piersol (2000):

$$\rho_{yy_{sim}} = \frac{\sum_{i=1}^{N} \left[\left(y_{i}(t) - \frac{1}{N} \sum_{i=1}^{N} y_{i}(t) \right) \left(y_{sim_{i}}(t) - \frac{1}{N} \sum_{i=1}^{N} y_{sim_{i}}(t) \right) \right]}{\sqrt{\sum_{i=1}^{N} \left[\left(y_{i}(t) - \frac{1}{N} \sum_{i=1}^{N} y_{i}(t) \right)^{2} \right]} \sqrt{\sum_{i=1}^{N} \left[\left(y_{sim_{i}}(t) - \frac{1}{N} \sum_{i=1}^{N} y_{sim_{i}}(t) \right)^{2} \right]}$$
(21)

can be used to estimated how well the estimated signals can reproduce the measured data. If the correlation coefficient is close to unity, one may conclude that the estimation algorithm can provide a good fit to the experimental data, but on other hand, if the coefficient is close to 0, the estimation was poor.

The fitting index, represented by the normalized correlation coefficient, is depicted in Fig. 6. In this figure, each group of parameters represents the fitting index of the estimated outputs (v, p, ϕ and r), obtained with three different types of maneuver at same airspeed and altitude: (a) 3-2-1-1 sequence with both lateral cyclic and pedal inputs used to estimate the flight derivatives; (b) second 3-2-1-1 sequence with lateral cyclic inputs and pedal fixed; and (c) third 3-2-1-1 sequence with pedal inputs and lateral cyclic fixed. The (b) and (c) groups of fitting index correspond to complementary flight data used only to validate the estimated parameters.



Figure 6. Correlation function coefficient for each measured output

Considering the assumptions and limitations of the theoretical helicopter dynamic model, it can be concluded that the obtained results are satisfactory, reaching fitting coefficients for the observed response in excess of 60% for all of the cases, and superior to 80% for most cases, despite the lack of a priori knowledge about the stability and control derivatives of the vehicle. This conclusion is confirmed analyzing the time response of all parameters (even the lower coefficients don't have amplitude discrepancies greater than 5 kt and 5° for v and ϕ parameters, respectively) and the Cramér-Rao Lower Bound (CRLB). It expresses a lower bound on the variance of each parameter estimated and, as presented by Tischler and Remple (2006), normalized CRLB lower than 20% suggests a satisfactory estimation.

As all flight derivatives identified are dimensional, Tab. 1 presents the mass and inertia parameters used.

Table 1. AS 355-F2 inertia and mass data

| Weight (kg) | Ixx (kg.m ²) | Iyy (kg.m ²) | Izz (kg.m ²) | Ixz (kg.m ²) |
|-------------|--------------------------|--------------------------|--------------------------|--------------------------|
| 2200 | 3.5191e+003 | 1.1996e+004 | 8.6216e+003 | -3.7007e+003 |

The estimated aerodynamic derivatives of the AS355-F2 lateral-directional motion at 80 kt and 5,000 ft and the respective CRLB are given in Tab. 2.

| Flight Derivatives | Identified Parameters | CRLB | Normalized CRLB (%) |
|--|-----------------------|--------|---------------------|
| Y_{v} [1/s] | -0,2218 | 0,0071 | 3,2 |
| Y_p [m/s.rad] | -0,8051 | 0,1573 | 19,5 |
| Y_r [m/s.rad] | -1,6897 | 0,1396 | 8,3 |
| L'_{v} [rad/m.s] | -0,0508 | 0,0006 | 1,2 |
| L'_p [1/s] | -2,6332 | 0,0517 | 2,0 |
| L'_{r} [1/s] | 0,2696 | 0,0253 | 9,4 |
| N'_{v} [rad/m.s] | 0,0442 | 0,0004 | 0,9 |
| N'_{p} [1/s] | -0,6053 | 0,0341 | 5,6 |
| N'_{r} [1/s] | -1,2201 | 0,017 | 1,4 |
| $Y_{\delta A}$ [m/s ² .cm] | 0,8580 | 0,0864 | 10,1 |
| $Y_{\delta P}$ [m/s ² .cm] | -0,8089 | 0,0571 | 7,1 |
| $L'_{\delta A}$ [rad/s ² .cm] | 0,4121 | 0,007 | 1,7 |
| $L'_{\delta P}$ [rad/s ² .cm] | -0,0216 | 0,0035 | 16,2 |
| $N'_{\delta A}$ [rad/s ² .cm] | 0,0674 | 0,0056 | 8,3 |
| $N'_{\delta P}$ [rad/s ² .cm] | 0,2821 | 0,0025 | 0,9 |

Table 2. Stability and control derivatives of the uncoupled lateral-directional mode at 80 kt and 5,000 ft

The time delay τ , the bias vector \dot{x}_{bias} and the constant vector y_{ref} were also identified. Their values are written in Tab. 3. The time delay, as shown by Eq. (6), was identified as the shift of the outputs measurements that minimizes the cost function given by Eq. (13).

| Parameter | Identified Value |
|--------------------------------|------------------|
| <i>v̇_{bias}</i> [m/s] | 0,1408 |
| \dot{p}_{bias} [rad/s] | 0,1960 |
| $\dot{\phi}_{bias}$ [rad] | 0,0063 |
| \dot{r}_{bias} [rad/s] | 0,0030 |
| v_{ref} [m/s] | -0,1037 |
| p_{ref} [rad/s] | -0,0034 |
| ϕ_{ref} [rad] | -0,0034 |
| r_{ref} [rad/s] | -0,0029 |
| $\tau_{\delta A}$ [s] | 0,0100 |
| $\tau_{\delta P}[s]$ | 0,0100 |

Table 3. Other Identified Parameters at 80 kt and 5,000 ft

7. CONCLUDING REMARKS

From the methodology M^4V , it was possible to obtain the uncoupled lateral-directional stability and control derivatives in forward flight of the AS355-F2 helicopter at 80 kt and 5000 ft.

The results showed that the genetic and Levenberg-Marquardt combined optimization algorithms are powerful tools for parameter estimation problems by the output-error method, especially, when the level of information regarding the studied system is reduced. As stated at previous sections, the adopted methodology initially estimates the flight derivatives using the genetic optimization algorithm and then, when the cost function tends to converge, the parameters are improved using Levenberg-Marquardt algorithm. This process joins the mean advantages of each algorithm:

• genetic optimization algorithm: guides the parameters along the right path, despite the lack of a priori knowledge about the stability and control derivatives of the vehicle, avoiding divergences and results without physical meaning, since GA facilitates the introduction of constraints;

• Levenberg-Marquardt optimization algorithm: guide the parameters to a precise local minimum and also requires less computational time.

Concerning flight test maneuvers, the 3-2-1-1 sequence was suitable to identify the uncoupled lateral-directional stability and control derivatives at 80 kt and 5,000 ft.

However, the helicopter dynamics has non-linear terms, with parameters that may vary abruptly with the modification of the flight conditions, or with helicopter configurations (large flap hinge-offsets), which require a larger number of degrees-of-freedom in the model to properly introduce the rotorcraft dynamics. Thus, in spite of satisfactory

results obtained with linear models for the lateral-directional dynamic mode, more complex models will be required to improve the curve fitting procedures. This is especially relevant to obtain level D flight simulator models and to develop control system strategies that depend on a comprehensive knowledge of the rotor and fuselage dynamic interaction in the entire flight envelope (weight, CG, altitude and airspeed) of the rotorcraft.

For the next studies, the following points will be taken in consideration:

• A more complex model will be proposed to include both longitudinal and lateral-directional dynamics modes;

• A improved model with at least 9 DOF in order to compute the coning angle and first harmonic coefficients of lateral and longitudinal blade flapping with respect to shaft;

• Application of different optimization techniques, such as Extended Kalman Filter (EKF), in conjunction with the maximum likelihood estimation problem; and

• Application of optimization techniques to design maneuvers in order to maximize the level of information contained in the flight test data and, consequently, to make the stability and control derivatives estimation more robust and precise.

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9. RESPONSIBILITY NOTICE

The authors are the only responsible for the material included in this paper.