2009 Brazilian Symposium on Aerospace Eng. & Applications
3rd CTA-DLR Workshop on Data Analysis & Flight Control
September 14-16, 2009, S. J. Campos, SP, Brazil

APPLICATION OF A STOCHASTIC IN-FLIGHT THRUST DETERMINATION PROCESS TO REAL ENGINE DATA

João Carlos C. Hoff, Ph.D.
jhoff@uol.com.br
João Roberto Barbosa, Ph.D.
barbosa@ita.br
Gas Turbine Group
Instituto Tecnológico de Aeronáutica. Praça Marechal Eduardo Gomes 50, 12228-900 São Jose dos Campos, SP, Brazil

Abstract. *Traditionally jet engines in-flight thrust determination is a deterministic process that starting from the fan and core measured pressures and temperatures calculates, by many different ways, the intermediate turbo machine parameters up to the exhaust nozzle pressure and temperature while iteratively search for the engine mass flow, attending simultaneously energy, mass and momentum conservation. With this data and the nozzle coefficients, determined from nozzle model tests, the installed thrust is calculated. A new approach was recently proposed by: (1) extending the concepts of the Residual Error Method, and (2) by applying a more stochastic approach, which in fact estimates iterativaly, from initial values, the air mass flow and the engine gross thrust values by minimizing the error between the fan and core calculated and measured air pressures and temperatures using the Output-Error Method and a modified Newton-Raphson minimization algorithm. The advantages of the new technique over the traditional one is that it has stochastic characteristics allowing to process the noise flight test data samples without previous data averaging over the test point stabilization time interval. This paper presents the application of the new technique to a set of real engine data recorded in-flight, showing agreement with traditional methods.*

Keywords*: In-Flight Thrust Measurement, Output-Error, Residual Error Method.*

NOMENCLATURE

A	Nozzle area, m^2
Ap	Area pressure method
ATF	Altitude Test Facility
C	Unknowns of the identification process
Cd	Nozzle mass flow coefficient, dimensionless
Cv	Nozzle thrust coefficient, dimensionless
R	Covariance of residuals
Deck	Engine cycle model
EREM	Extended Residual Error Method
FAR	Federal Air Regulation
FMU	Fuel Management System
FPC	Fan Pressure Correlation, dimensionless
FTB	Flying Test Bed
FTI	Flight Test Instrumentation
FG_c	Calculated gross thrust from GLTF data, *N*
FG_m	GLTF measured gross thrust, *N*
FNet	Net thrust, *N*
GLTF	Ground Level Test Bed
IFTD	In-flight thrust, determination
N	Samples number
N	Newton (force unit)
Pa	Pascal, pressure unit
Pamb	Ambient pressure, *Pa*
Pt14c	Total pressure at station 14, *Pa*
REM	Residual Error Method, dimensionless
RERR	Residual of the Residual Error
SAE	Society of Automotive Engineers
SFC	Specific fuel consumption, *kg/h/KN*
W_c	Calculated air mass flow from GLTF data, *kg/s*
W_m	GLTF measured air mass flow, *kg/s*

1. INTRODUCTION

The development and certification of an aircraft requires the estimation of the engines installed thrust for the drag evaluation and performance calculation. For transport category airplanes, the FAR 25 (1999), states: "*the performance must correspond to the propulsive thrust available under the particular ambient atmospheric conditions, the particular flight condition, and the relative humidity specified in the regulation. The available propulsive thrust must correspond to the engine power or thrust, not exceeding the approved thrust less:*

1. *Installation losses, and*
2. *The power or equivalent thrust absorbed by the accessories and services appropriate to the particular ambient atmospheric conditions and particular flight condition."*

To comply with the above requirements while verifying simultaneously the actual engine SFC specified in contracts with engine manufacturer, the airframe manufacturers use to measure in-flight the installed engine thrust and fuel flow. The measurements are performed for a range of altitude, airspeed, fan speed, and power extraction and the results are used for calibration of the engine Deck (cycle model). Later, this cycle model becomes the general source of installed thrust for flight test data analysis and airplane certified performance calculation.

Jet engine installed thrust is indirectly estimated by processes that require the engine nozzle characterization from nozzle model tests in test rig, and the measurement of actual engine pressures and temperatures, at the same stations measured in the model during nozzle model characterization. The literature presents several methods of determining deterministically the installed thrust. The reports SAE AIR 1703A (2006) and SAE AIR 5450 (2006) are the guides on the subject reflecting the state of the art and industry standards. Among the SAE presented methods the more accurate and used by the industry are the denominated $W\sqrt{T}$ (Weight temperature method), Ap (Area pressure method) and the Residual Error Method (REM), all '*gas flow path*' type methods.

The nozzle calibration process consists in determining in a test rig the nozzle coefficients Cv, Cd (or others) for the nozzle scale model. The nozzle coefficients are dimensionless groups that relate the actual measured nozzle thrust to the ideal calculated nozzle thrust and the actual measured mass flow to ideal calculated mass flow. In-flight the ideal thrust and mass flows are calculated from the engine measured parameters, and then, via the nozzle coefficients, the actual thrust and mass flow are estimated. Before this step it is required to calculate the nozzle total pressure and total temperature and / or others, which may be calculated from turbo machine maps, energy, mass and momentum conservation, and will be used in mass flow and gross thrust calculation.

In-flight thrust estimation requires a refined plan, an accurate and expensive instrumentation, nozzle model calibration, real engine calibration on GLTF, FTB, ATF, several hours of expensive flights and many hours of engineering analysis.

This work deals with the inverse or backward formulation of the in-flight thrust estimation process published by Hoff (2007), that is, starting from initial values of nozzle parameters as temperature, pressure and air mass flow, the fan and the core total pressure, temperatures and the engine fuel flow are calculated. The error between the calculated and the measured parameters are minimized via an optimization algorithm by appropriately inserting the values of gross thrust (or nozzle total pressure) and air mass flow (fan and core). The method is applied to a separate-stream large turbofan using real engine data.

2. DEVELOPMENT

2.1 The residual error method

The Residual Error Method (REM), presented in SAE AIR 1703A (2006), has been developed to overcome the problem of testing very large turbofan engines in ATF, that is of difficult logistics and very expensive. When an engine is tested on a GLTF the thrust and flow coefficients obtained in the model calibration may not be reproduced, i.e., the model calibrated in the test facility does not match accurately the real engine. In addition, the range of pressure ratios achieved in ground tests do not cover the required range achievable in-flight so that the model data shall be considered. One way found to overcome this problem was recalibrating the model in a way that it reproduces the GLTF results using the nozzle model coefficients, and this has been done for large turbofans by searching for a multiplier (a real number) that applied to the fan pressure makes the model match more closely the GLTF results while minimizing the following cost functional, Eq. (1):

$$REM = \sqrt{\left(\frac{FG_c}{FG_m} - 1\right)^2 - \left(\frac{W_c}{W_m} - 1\right)^2} \quad (1)$$

This multiplier has been denominated Fan Pressure Correlation (FPC) and the method assumes that error exist in the pressure measurements. Using the GLTF data a set of FPC is determined for the range of engine pressure ratio. Figure 1 shows the minimization of the Residual Error for one test point.

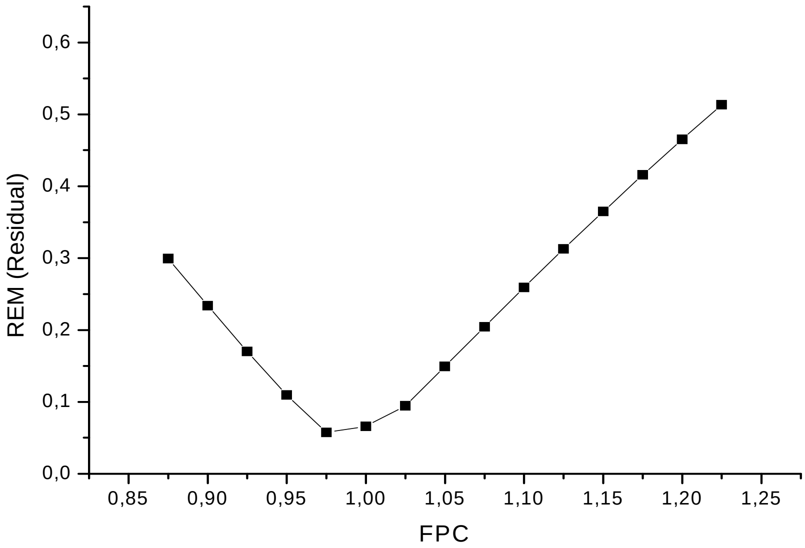

Figure 1. Minimizing the Residual Error via FPC

For in-flight thrust determination, the FPC is used to multiply the fan pressure and, based on engine manufacturers observations, the processes improves the accuracy of the thrust calculation allowing to discard the use ATF tests.

The Extended Residual Error Method (EREM) has been introduced by Hoff (2007) and is applicable to long cowl engines with mixer. However, its use may be the extended also to other engine types. It allows for the calculation of other parameters in addition to the FPC, as for instance a bias to the mixer temperature, in addition to the errors in pressure measurements assumed by the standard REM. For engines with mixer it is assumed a model to the mixer pressure and temperature, sources of error in the thrust modeling process. The FPC may also reduce the error due to the pressure modeling but is not sufficient to compensate for temperature modeling errors. The EREM has been formulated using the Output-Error over the nozzle characterization tests and actual engine tests in GLTF and the cost functional of Eq. 1, that is, the same of the standard REM. The optimization process is analogous to the one presented in the section 2.2 below.

2.2 Stochastic in-flight thrust estimation process

It is applied an optimization algorithm for estimating in-flight the installed engine gross thrust and air mass flow by minimizing the error between calculated and measured engine parameters – *Output-Error* concepts. The problem is formulated as an optimization problem as published by Hoff (2007) where given the measured quantities as fan and core pressures, temperatures, fuel flow, etc., the algorithm inserts values of gross thrust, fan and core air mass flow, by minimization of a cost functional based on the calculated and measured fan and core pressures, temperatures, fuel flow, etc. The cost functional (Eq. 2) comes from the negative logarithm of the *Likelihood Function,* which is derived from the probability density function of a Gaussian process whose minimization is analogous to maximize the conditional probability $p(c/Z)$.

$$J = \frac{1}{N-1}\sum_{K=1}^{N}(z-\hat{x})^T R^{-1}(z-\hat{x}) \quad (2)$$

In (2) z are the measured parameters (Tt13, Pt13, Tt49, Pt49) and $\hat{x}$ are the equivalent values estimated in the inverse process.

Equation (2) is here minimized by a Newton-Raphson type algorithm and the unknowns are calculated iteratively by Eq. (3) from given initial values:

$$c_L = c_{L-1} - \left[\frac{\partial^2 J}{\partial c^2}\right]^{-1} \frac{\partial J}{\partial c} \tag{3}$$

For example, in (3) the values for Ts and Mass Flow are used as the optimization parameters for an engine with mixer, while Ptnozzle and Ttnozzle for engines with separate flow.

The derivatives of J to the unknowns are calculated by Eq. (4):

$$\begin{aligned} \frac{\partial J}{\partial c} &= \frac{2}{N-1}\sum_{k=1}^{N}(z-\hat{x})^T R^{-1}\frac{\partial (z-\hat{x})}{\partial c} \\ \frac{\partial^2 J}{\partial c^2} &= \frac{2}{N-1}\sum_{k=1}^{N}\frac{\partial (z-\hat{x})^T}{\partial c} R^{-1}\frac{\partial (z-\hat{x})}{\partial c} \end{aligned} \tag{4}$$

where $\frac{\partial (z-\hat{x})}{\partial c}$ are partial derivatives that are calculated numerically using finite differences. To do so, the optimization parameter c is varied of 1% relative to its nominal value in the present iteration. Since z is measured and constant, what implies $\partial z = 0$. In the other hand, x are the temperatures and pressures equivalent to the measurements (z). Therefore, the derivatives state, as x vary relatively to the optimization parameters, namely the nozzle pressures and temperatures or the mass flow. It is important to note that $(z-\hat{x})$ is a vector; R is a diagonal matrix with the same number of rows as $(z-\hat{x})$; the partial derivatives are matrices; the first derivative of J with respect to c is a vector.

In Eq. (2) and (4) above, R is the matrix of the covariance of the residuals which is calculated by Eq.(5):

$$R = \frac{1}{N-1}\sum_{K=1}^{N}(z-\hat{x})(z-\hat{x})^T \tag{5}$$

R may be initialized with ones (1) in its main diagonal. The thrust can be calculated by any standard method that is appropriate for backwards formulation. Since the data processing is carried out backwards, precision is the same as the IFTD standard method, not the algorithm's. The backwards process, in many cases, may produce unknowns not present in the standard direct process, but these unknowns are determined simultaneously during the iterative process, by formulating as a measured pseudo-parameter. Example may be an equivalent area that must be equal to a nominal one. One pretends that a nominal area is a measure and one minimizes the error between the equivalent and the nominal area. Similarly, unknown mixer parameters may be calculated when the process is applied to a mixed turbofan engine. The optimization algorithm used in this research is widely used among system identification community.

The calculations may be carried out in the following order:

- Attribution of initial values for the optimization parameters
- Loop of optimization
- Loop of the flight measurements
- Loop for the calculations with the initial values and the perturbed initial values
- End of previous loop, from which all states are calculated, with and without perturbations
- Calculation of the partial derivatives with central differences

$$\frac{\partial x(I)}{\partial c(J)} = \frac{X(I,J) - X(I,J+1)}{\Delta c(J)}$$

- Calculation of the cost functional first and second order partial derivatives, equation (4)

$$\frac{\partial J}{\partial c} = \frac{2}{N-1}\sum_{k=1}^{N}(z-\hat{x})^T R^{-1}\frac{\partial(z-\hat{x})}{\partial c}$$

$$\frac{\partial^2 J}{\partial c^2} = \frac{2}{N-1}\sum_{k=1}^{N}\frac{\partial(z-\hat{x})^T}{\partial c} R^{-1}\frac{\partial(z-\hat{x})}{\partial c}$$

- End of the flight measurement loop
- Inversion the matrix of the second derivative of J
- Actualization of the nominal values of the optimization parameters c.

$$c_L = c_{L-1} - \left[\frac{\partial^2 J}{\partial c^2}\right]^{-1}\frac{\partial J}{\partial c}$$

- End of the optimization loop. Repeat the loop until a specified convergence criterion is fulfilled, say, the differences of consecutive values are irrelevant, or J has converged.

The estimates accuracy may be evaluated by the Cramér-Rao lower bound (variance theoretical minimum values), which can be calculated from terms of Eq. (3), or by its improved version for *colored* noise as presented in Morelli, Klein (1994). However, the overall accuracy of the estimated parameters is dependent of the thrust method used behind the optimization process. The Cramér-Rao bound reflects in fact how good was the insertion of the estimated parameters for the available flight data and reflects more a characteristic of the data set.

The algorithm allows processing a set of test samples recorded in-flight during an engine/airplane stabilization taking into account the noise of the recorded data. To demonstrate the use of the estimation process an in-flight thrust algorithm applicable to a separate-stream turbofan has been chosen and the deterministic (traditional) and the new probabilistic process are compared.

3. RESULTS

This section presents the application of the above described processes to a set of GLTF data and to a set of real engine in-flight recorded data, the case study.

In Residual Error Method analysis all GLTF data are processed for determining a curve of fan pressure ratio versus the fan pressure correlation, for the various instrumented test engines that will be used in the test campaign.. For the case study only one FPC is needed for pressure ratio of the recorded in-flight data. In order to have a more representative value of FPC five GLTF test points, whose fan pressure ratio are close to the test case, have been processed.

Since there is only an average value of the calculated and measured values of thrust and mass flow, fifth samples have been created by adding noise to the GLTF measured thrust and mass flow. Noise has been created in Matlab^R using observed values of standard deviation of calculated thrust and mass flow for the same engine. No additional terms as in the EREM has been considered in order to compare the results of the probabilistic process with the previously calculated deterministic process. For the five GLTF available points, the FPC was calculated using the algorithm given by Eq. (2), (3), (4), (5) and the of Cv and Cd curves available from nozzle model characterization lab tests.

Table 1 – Residual Error Method Application

Point	FNet – meas. (N)	FNet – calc. (N)	W2 – meas. (kg/s)	W2 – calc. (kg/s)	REMR -	Pamb (pa)	Ptfan (pa)	FPC -
1	65591	65618	192.64	192.51	0.0008	98612	174782	0.979
2	65147	65160	192.09	192.03	0.0004	98604	174368	0.980
3	64378	64390	191.20	191.15	0.0004	98597	173541	0.980
4	62785	62777	189.17	189.21	0.0002	98593	171748	0.981
5	60797	60746	186.17	186.42	0.0016	98579	169335	0.981

Table 1 presents the results of the application of the REM to the GLTF data. For the case study, a FPC of 0.98 will be used in both the deterministic and probabilistic processes.

One set of flight data (30 sec of data at 10 sps) has been processed by the deterministic and the probabilistic methods. For the probabilistic method the data has been averaged while for the probabilistic algorithm the data has been processes sample by sample along the time. Figure 2 presents the traces of two parameters of the data sample. The noise level is reasonably low.

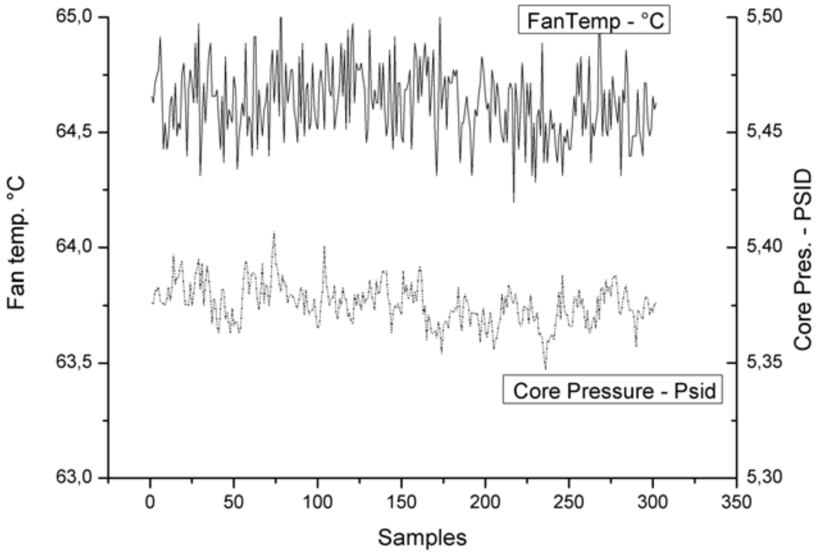

Figure 2. Fan total temperature and core differential pressure – typical traces

Figures 3.and 4 present the By-pass mass flow and the By-pass nozzle static temperature calculated by the probabilistic method. It is visible the easy convergence of the parameters in approximately 8 iterations.

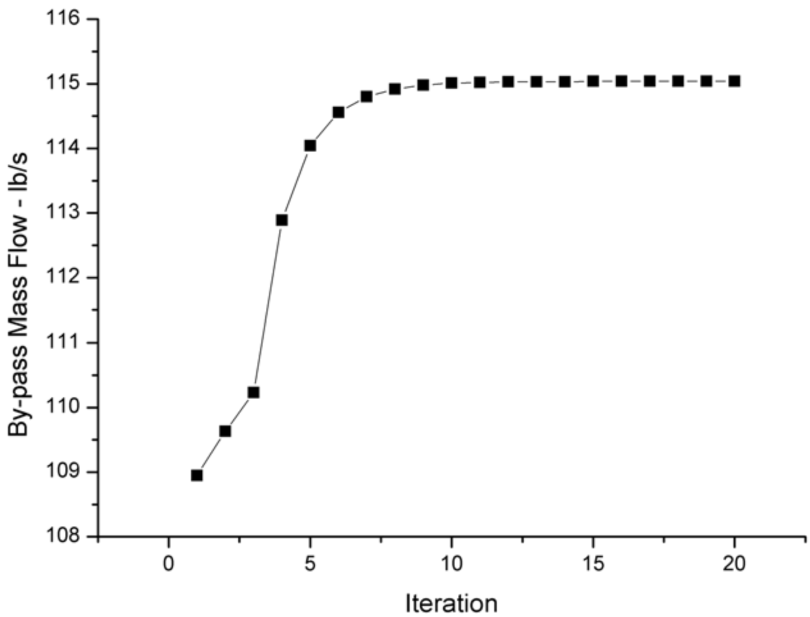

Figure 3. By-pass Mass Flow

Figures 5 and 6 present the core mass flow and static temperature calculated by the probabilistic method where is visible the easy convergence of the parameters in approximately 8 iterations.

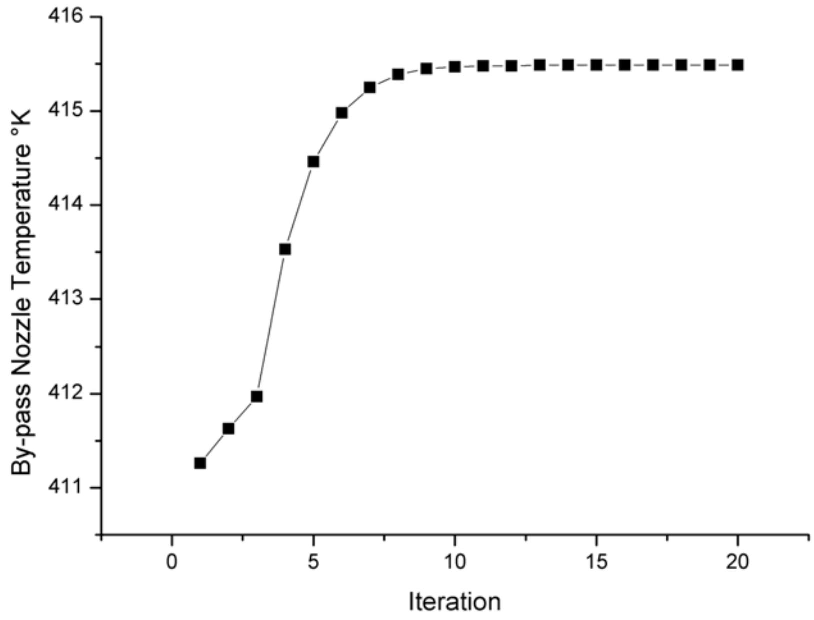

Figure 4. By-pass Nozzle Static Temperature (Ts19)

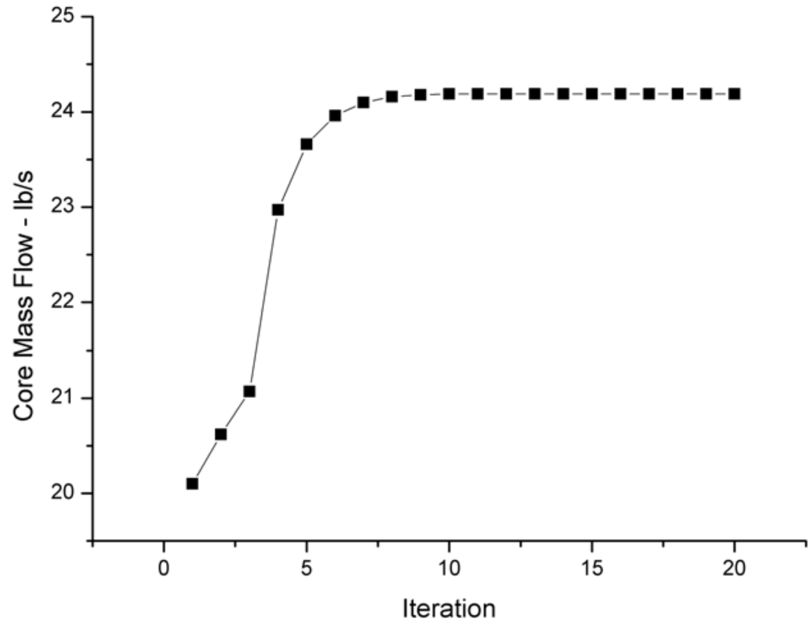

Figure 5. Core Mass Flow

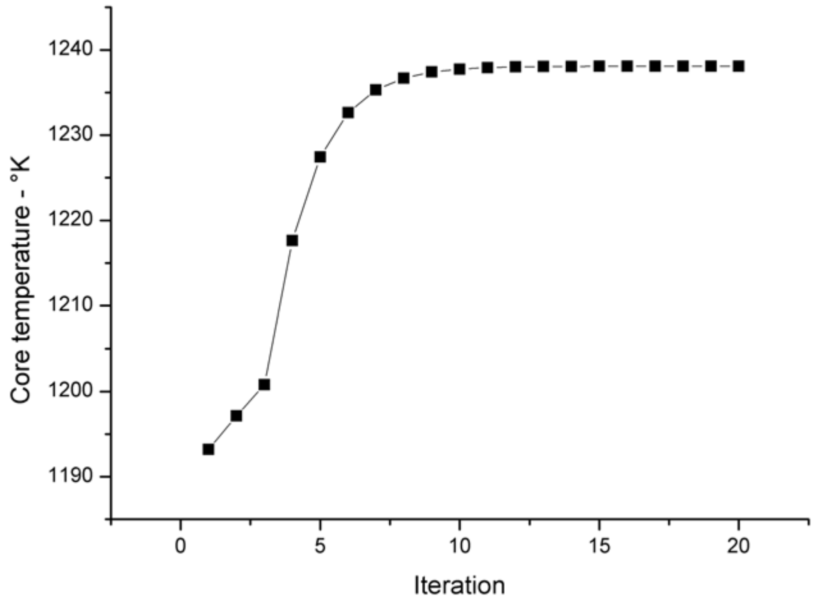

Figure 6 – Core Static Temperature (Ts9)

Table 2 presents the resultant thrust and mass flow from the deterministic and probabilistic methods. The deterministic method is analogous to the method presented under 6.6.5 of report SAE AIR 1703A, while the probabilistic is the backward application of the same method, what allows a direct comparison of the two techniques. Observe that the two techniques present approximately the same results. Differences are due to the impact of averaging or not averaging, since, for instance, an outlier produces more impact on the deterministic method than on the probabilistic. In the later, as it works with the standard deviations of the errors, the differences associated with outliers are less weighted than in the case of averaging.

Table 2 – Calculated Thrust and Mass Flow

	Direct Algorithm	**Stochastic Algorithm**
Net Thrust (N)	**11564**	**11559**
FG19 (N)	**18502**	**18547**
FG9 (N)	**6387**	**6396**
Fan Mass Flow (kg/s)	**51.5**	**51.6**
Core Mass Flow (kg/s)	**11.05**	**11.03**

4. FINAL COMMENTS

A statistical approach to in-flight thrust estimation for a large separate-stream turbofan engine has been demonstrated for flight test data. The new process finds the unknowns thrust and air mass flow, through an optimization process, minimizing the error between calculated and measured parameters.

The main advantage of the process is that it allows for the use of a large set of data samples without previous time averaging. As a statistical process the resultant thrust and air mass flow reflects the properties associated to the whole data set and is updated iteratively by the optimization process. It also allows using data from different sources, as for instance fuel flow data from FTI and from FMU, from both FADEC channels, simultaneously in the optimization process. The nozzle characterization, that is, the determination of nozzle coefficients from nozzle model tests, shall take into account a direct process equivalent to the inverse problem.

The algorithm is robust and may be started with mass flow from generalized flow function for the fan and core not requiring good initial estimates. Disturbing the initial values the algorithm it is able to converge always to the same solution. As an optimization process the algorithm allows for different implementations depending on the constraint equations and optimization parameters used in the problem formulation. For the present application the use of the air mass flow and the nozzle static temperature as optimization parameters resulted in the best algorithm implementation among the tested configurations. For long cowl engines with mixer the optimization parameters may be the total pressure and temperature.

The algorithm is not too fast as the deterministic algorithm. For the presented example three hundred samples have been used in the calculation (30 seconds of flight data) taking around 15 seconds to produce the results. The partial derivatives required by Eq. 4 were numerically calculated by central differences, from perturbations to the optimization parameters. A 1% perturbation produced good results for the chosen optimization parameters. However, it may be a good practice to perform an additional iteration with 0.5% perturbation to verify the influence of partial derivatives accuracy.

It has been observed that processing all the sensors samples or averaging all sensors data for a instant of time do not affect significantly the final results and the following approach has been considered: process the data averaged by rake and only after having reached a result perform one or two more iteration using all the samples.

The technique does not reflect a new in-flight thrust estimation process different from those SAE standards. It is in fact a new way of processing data from any standard method formulated backwards. However, the technique opens the possibilities of implementing in-flight thrust estimation as stochastic filtering.

5. REFERENCES

PART 25, Feb. 8, 1999. *Airworthiness Standard: Transport Category Airplanes*. FAA, Department of Transport, USA.

SAE AIR 1703A, Feb. 2006. *In-Flight Thrust Determination*. Society of Automotive Engineer. USA.

SAE AIR 5450, Draft 2006-06-16. *Advanced Ducted Propulsor In-Flight Thrust Determination* Society of Automotive Engineers. USA.

Morelli, E, A; Klein, V. 1994. *Determining the accuracy of Maximum Likelihood parameter estimates with colored residuals*. NASA Report 194893.

Hoff, J. C. Nov. 2007 a. *The Extended Residual Error Method for Jet Engines In-flight thrust Determination*. COBEM 2007, Brasilia, DF.

Hoff, J.C. Nov. 2007 b. *Towards a Stochastic In-Flight Determination Process*. Paper SAE 2007-01-2542. SAE BRASIL 2007 Congress, São Paulo.

7. RESPONSIBILITY

The authors are the only responsible for the printed material included in this paper.