USING LOW THRUST PROPULSION TO ORBIT CONTROL OF THE LUNAR POLAR SATELLITES

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Abstract. *It is well known that lunar satellites in polar orbits suffer a high increase on the eccentricity due to the gravitational perturbation of the Earth. That effect is a natural consequence of the Lidov-Kozai resonance. The final fate of such satellites is the collision with the Moon. Therefore, the control of the orbital eccentricity leads to the control of the satellite's lifetime. In the present work we study this problem and introduce an approach in order to keep the orbital eccentricity of the satellite at low values. The whole work was made considering two systems: the 3-body problem, Moon-Earth-satellite and the 4-body problem, Moon-Earth-Sun-satellite. First, we simulated the systems considering a satellite with initial eccentricity equals to 0.0001 and a range of initial altitudes between 100km and 5000km. In such simulations we followed the evolution of the satellite's eccentricity. We also obtained an empirical expression for the length of time needed to occur the collision with the Moon as a function of the initial altitude. The results found for the 3-body model were not significantly different from those found for the 4-body model. Secondly, using low thrust propulsion, we introduced a correction of the eccentricity every time it reached the value 0.05. These simulations were made considering a set of different thrust values, from 0.1N up to 0.4N which can be obtained by using Hall Plasma Thrusters. In each run we measured the length of time, needed to correct the eccentricity value (from e = 0.04 to e = 0.05). From these results we obtained empirical expressions of this time as a function of the initial altitude and as a function of the thrust value.*

Keywords: *lunar polar orbit, kozai resonance, low thrusters propulsions*

1. INTRODUCTION

Recently, several nations presented plans to reach the Moon. Satellites have been launched and many more are planned for following years (see for instance Foing & Ehrenfreund (2008). The expectations are that in the near future there will be a lunar base. The lunar poles are particularly of interest since seems to be where water can be found. Therefore, long living satellites in polar lunar orbits will be needed. It is well known that lunar satellites in polar orbits suffer a strong gravitational perturbation from the Earth. That effect is a natural consequence of the Lidov-Kozai resonance.

It is well known that the Lidov-Kozai resonance introduces equilibrium configurations. In the case of lunar polar orbits disturbed by the Earth's gravitational field, this can be used as an advantage to implement constellations of satellites with elliptic highly inclined orbits (Ely (2005); Ely & Lieb (2006)). On the other hand it causes instability for near circular highly inclined orbits. Wytrzyszczak et al. (2007) studied the regular and chaotic motion of geosynchronous satellites disturbed by the Moon's gravitational field. They found that the chaotic nature of high inclination satellites is caused due to the significant eccentricity growth caused by the Lidov-Kozai resonance.

Similarly, the final fate of polar lunar near circular satellites is the collision with the Moon. Therefore, the control of the orbital eccentricity leads to the control of the satellite's lifetime.

In this paper we purpose the control of the eccentricity, using a electrical thruster, similar of that is in development at the University of Brasília. Electric propulsion is basically a technique of space propulsion which evolves the conversion of electrical power into the kinetic power or thrust of the exhaust beam of ionized particles. The ability to obtain high exaust velocities with ionized particles enables plasma thrusters o perform high specific impulse mission in space (Ferreira, 2008). The main goal of the thruster that is being developed at the University of Brasília is the use of a permanent magnet, saving energy during the mission. Preliminary results in the laboratory show that is possible to obtain more than 100mN with this technology. Inspired on this thruster project, in this work, we assume a constant exhaust velocity, and we can control the switch of the thruster during the mission.

In the present work we study this problem and introduce an approach in order to keep the orbital eccentricity of the satellite at low values. The approach is based on the use of low thrust propulsion in order to introduced a correction of the eccentricity.

In the next section we introduce the Lidov-Kozai resonance. In section 3 we show the evolution of the eccentricities form our numerical integrations. The approach proposed to control the eccentricity and its application is presented in section 3. In the final section we present our final comments.

2.THE LIDOV-KOZAI RESONANCE

Lidov (1961), studying the dynamics of artificial satellites, and Kozai (1962), studying the dynamics of asteroids, independently discovered what is know called the Lidov-Kozai resonance. Following, we introduce the basic features of such resonance.

In this section we addopted a simple model (see for example Ely (2005)) for the orbital evolution of an artificial satellite disturbed by a third body in circular equatorial orbit around the primary. It was obtained by double averaging the system (Prado, 2003) taking into account the disturbing function expanded in Legendre polynomials up to second order and the eccentricity of the disturbing body also up to the second order. The disturbing function of the problem is averaged independently over the mean longitudes of the satellite and the third body. The standard definition for average used in this work is:

$$\langle F\rangle=\frac{1}{2\pi}\int_0^{2\pi}(F)\,dM \tag{1}$$

where M is the mean anomaly, which is proportional to the time.

Following such approach one can find the double averaged disturbing function given by (see for example Vashkov'yak & Teslenko, 2007):

$$R=\frac{3G(m_1+m_2)a^2}{16a_E^2}\left(2\left(e^2-\sin^2 i\right)+e^2\left(5\cos 2\omega-3\right)\sin^2 i\right) \tag{2}$$

where a, e, ω and i are respectively the semi major axis, eccentricity, argument of pericenter and inclination, G is the gravitational constant, a_E is the semi-major axis of the Earth with respect to the Moon, m_1 and m_2 and are the masses of the Earth and Moon respectively. Substituting R in Lagrange's planetary equations (see for example Kovalevsky (1967)), we find:

$$\frac{da}{dt}=0 \tag{3}$$

$$\frac{de}{dt}=\frac{15e\gamma\sin r^2 i\sin(2\omega)\sqrt{1-e^2}\,n_E^2(2+3e_E^2)}{16n} \tag{4}$$

$$\frac{d\omega}{dt}=-\frac{3\gamma(2+3e_E^2)n_E^2(-5\cos 2\omega+5e^2\cos 2\omega+5\cos 2\omega\cos^2 i-5\cos^2 i+1-e^2)}{16n\sqrt{1-e^2}} \tag{5}$$

where n is the mean motion and $\gamma = m_1/(m_1+m_{12})$.

Considering the case when de/dt=0 and $d\omega/dt$=0 one can find three first integrals:

$$a=a_0 \tag{6}$$

$$(1-e^2)\cos^2 i=k_1 \tag{7}$$

$$e^2(2/5-\sin^2 i\sin^2 w)=k_2 \tag{8}$$

where a, e, i and w are the semi-major axis, eccentricity, inclination and argument of pericentre of the satellite, and a_0, k_1 and k_2 are the constants of motion. This system has a set of fixed points given by

$$w=90^o \quad \text{or} \quad 270^o \quad \text{and} \quad e^2+(5/3)\cos^2 i=1 \tag{9}$$

Therefore, for a system with e, i and ω satisfying conditions (9), the satellite would be in what can be called a frozen orbit, i.e., apart from short period oscillations, the orbit would be kept fixed in size and location.

A simple analysis of Equations (2) and (3) shows that (Vashkov'yak & Teslenko, 2007):

- for $k_2 > 0$ and any value of k_1: w circulates;
- for $k_2 < 0$ and $k_1 < 3/5$: w librates around 90° or 270° ;
- for $k_2 = 0$ and w = 90° or 270°: $i = i^* \sim 39.2°$;

So, that is the Lidov-Kozai resonance. When $i > i^*$, the system behaves like a pendulum, with stable fixed points, librations around such points and circulation. In Figure 1 we reproduce the results from Ely & Lieb (2006). It presents a sample of the satellite's orbital evolution in a diagram e versus w. It shows a the clear dependence of the eccentricity on the argument of pericentre for an orbit with high inclination. All satellites inclined to the orbital plane of the third body (the perturber) by more then a critical angle, i^*, experience a considerable growth of eccentricity. The third body causes the Lidov-Kozai resonance driving the eccentricity growth.

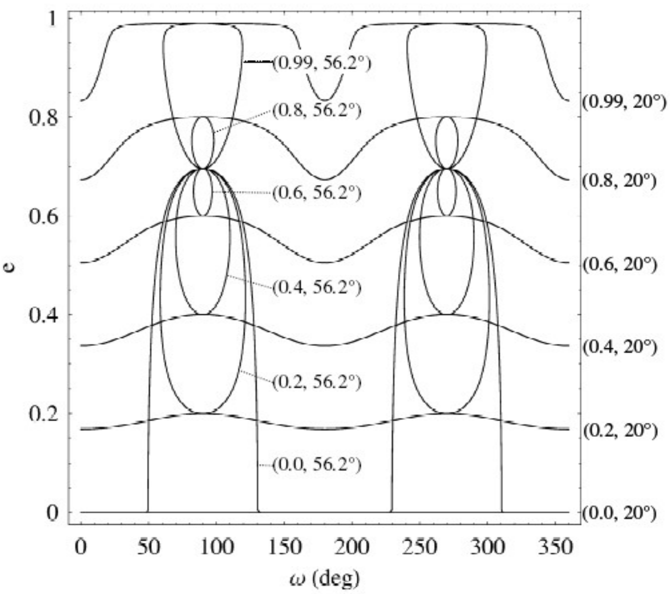

Figure 1. Sample of the satellite's orbital evolution in a diagram e versus w. There are two sets of initial values of eccentricity and inclination (e_o, i_o). One for low inclination $i_o = 20°$ and other for high inclination $i_o = 56.2°$. (reproduced from Ely & Lieb (2006)).

3. ECCENTRICITY GROWTH

In this section we present numerical simulations considering two dynamical systems: the 3-body problem, Moon-Earth-satellite and the 4-body problem, Moon-Earth-Sun-satellite. In all simulations the satellite is initially in polar orbit ($i = 90°$).

In the case of the 3-body problem, considering a coordinate system centered in the barycenter of the Earth- Moon system (X,Y,Z), the equations of motion of the satellite is given by:

$$\begin{aligned}
\ddot{\vec{x}} &= \sum_{i=1}^{2} G \frac{m_i}{|x_i - x|^3} (\vec{x}_i - \vec{x}) \\
\ddot{\vec{y}} &= \sum_{i=1}^{2} G \frac{m_i}{|y_i - y|^3} (\vec{y}_i - \vec{y}) \\
\ddot{\vec{z}} &= \sum_{i=1}^{2} G \frac{m_i}{|z_i - z|^3} (\vec{z}_i - \vec{z})
\end{aligned} \tag{10}$$

where m is mass and the index i=1 refer to the Earth and i=2 refer to the Moon. In this system, the equations of motion for the moon and the Earth are given by:

$$\begin{aligned}\ddot{\vec{x}}_i &= \sum_{j=1,\, j\neq i}^{2} G\frac{m_i}{|x_j - x_i|^3}(\vec{x}_j - \vec{x}_i)\\ \ddot{\vec{y}}_i &= \sum_{j=1, j\neq i}^{2} G\frac{m_i}{|y_j - y_i|^3}(\vec{y}_j - \vec{y}_i)\\ \ddot{\vec{z}}_i &= \sum_{i=1, j\neq i}^{2} G\frac{m_i}{|z_j - z_i|^3}(\vec{z}_j - \vec{z}_i)\end{aligned} \tag{11}$$

First, we simulated the system, integrating numerically the equations (10) and (11**),** considering a satellite with initial eccentricity equals to 0.0001 and a range of initial altitudes between 100km and 5000km. Figure 2 shows the evolution of the satellite's eccentricity for the 3-body simulations, considering altitudes h = 100, 200, 500, 1000 and 5000km. The plots show an exponential evolution of the eccentricity. We computed the time needed in order to reach the eccentricity that corresponds to the collision of the satellite with the Moon. A fit of the collision time, $T_{collision}$, as a function of the altitude, h, is given by the expression:

$$T_{collision} = 2693 \times e^{0.062\text{h} - 3.92\times 10^{-4} h^2} \tag{12}$$

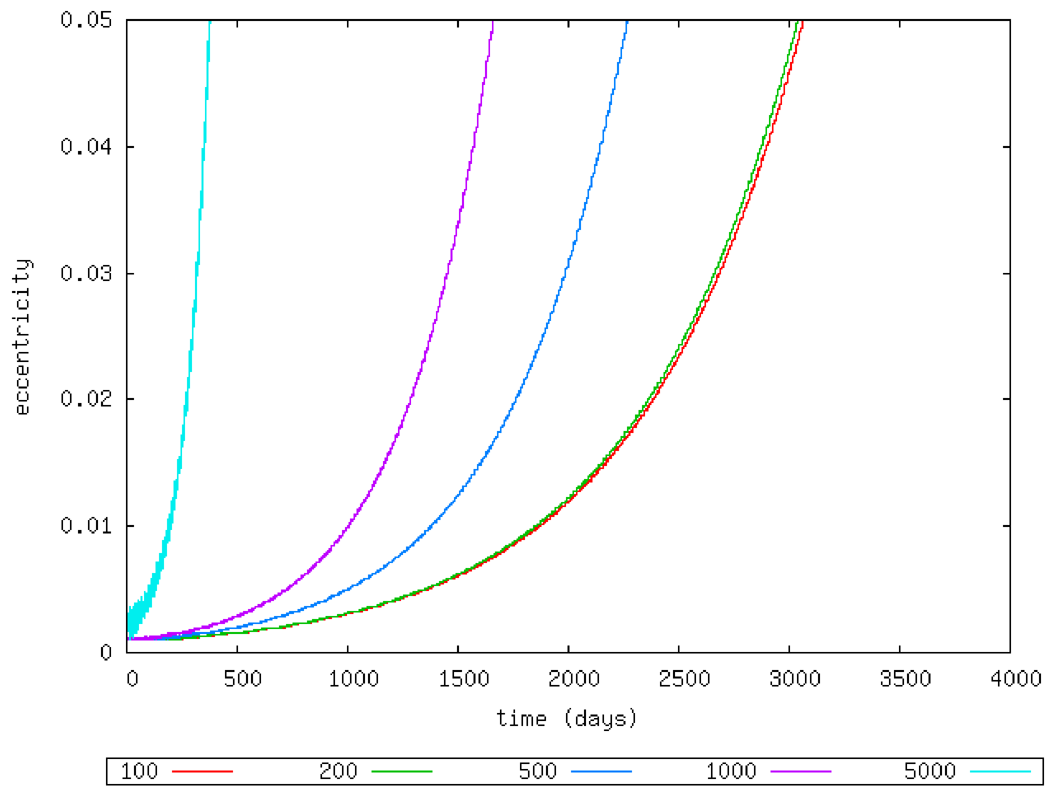

Figure 2. Time evolution of the eccentricity for the 3-body problem. The colour code indicates the initial altitude in kilometers.

The same set of simulations was performed considering the 4-body problem, adding the perturbations of the Sun. Now we considered a new coordinate system (X'Y'Z'), centered at the barycenter of Sun-Earth-Moon system. In this case we integrate numerically the equations similar (using prime in the variables) to equations A e B, but we add the index i=3, where the fourth term refer to Sun.

However, the results found for the 4-body model were not significantly different from those found for the 3-body model. The empirical expression for the length of time needed to occur the collision with the Moon as a function of the initial altitude is given by:

$$T_{collision} = 2494 \times e^{0.063h - 3.94\times 10^{-4} h^2} \quad (13)$$

A comparison of the two sets of simulations and the Equations (5) and (6) is shown in Figure 3.

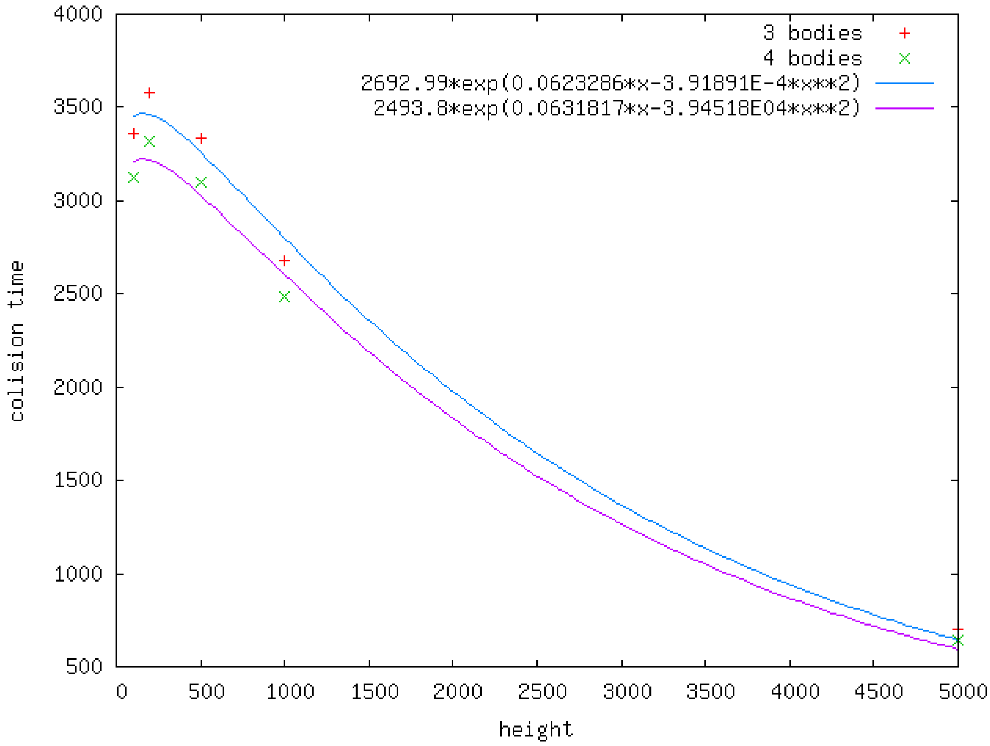

Figure 3. Collision time as a function of the initial altitude. The red crosses are for the 3-body simulations and green are for the 4-body simulations. The blue curve corresponds to Equation (5) and the purple curve corresponds to Equation (6).

4. CONTROLLING THE ECCENTRICITY

In order to control the satellite's eccentricity we will use low thrust propulsion. Following the work of Sukhanov, (2007), we use the locally optimal thrust for each orbital element. This development is based on the performance index, through the minimization of a functional in the direction of the orbital element to be changed. In our case, the eccentricity is the parameter to be minimized. The result is a vector, called Lawden's primer vector, P, which gives the direction of the thruster to be turned on.

The eccentricity of the satellite relative to the Moon is given by:

$$e = \sqrt{1 + \frac{c^2}{G m_2} h} \quad (14)$$

where c is the magnitude of the angular momentum, h is the integral energy. The primer vector is given by:

$$\vec{p} = \frac{1}{G m_2 e}\left(P \vec{v} - \frac{r^2}{a} \vec{v}_n\right) \quad (15)$$

where $\vec{r}$, $\vec{v}$ and $\vec{v}_n$ are the vector position, velocity and tangential velocity relative to the Moon, *a* and *P* are the semi-major axis and semi-latus rectum relative to the Moon

Then, we have that the acceleration components to change eccentricity are given by:

$$p_r = \frac{1}{\mu e} p_a \quad \text{and} \quad p_n = \frac{1}{\mu e}\left(p_a - \frac{r_p^2}{a}\right) \quad (16)$$

where $p_a = c^2/\mu$, p_r and p_n are the radial and the tangential components, respectively.

In order to perform the numerical simulations, we converted the accelerations given by equations (16) into the baricentric systems of coordinates for the 3-body problem (Earth-Moon-Spacecraft), then multiplied them by the thrust magnitude and, finally, added the respective components to the equations (10) and integrated together with equations (11).

The approach we are proposing is a very simple one. The idea is to introduced a correction on the eccentricity every time it reach a certain limit. The procedure is as follows. First fix the nominal eccentricity, e_o, and the maximum acceptable increase in eccentricity, Δe, according to the mission design. Then, turn on the thruster every time the condition

$$e > (e_o + \Delta e) \tag{17}$$

is satisfied and turn off the thruster when $e > e_o$.

Following, we present the results of some simulations assuming $e_o = 0.04$ and $\Delta e = 0.01$. These simulations were made considering a set of different thrust values, from 0.1N up to 0.4N. In each run we measured the length of time T_{Thruster}, needed to correct the eccentricity value (from $e = 0.04$ to $e = 0.05$). From these results we obtained empirical expressions of $T_{Thruster}$ as a function of the initial altitude and as a function of the thrust value. As an example, in Figure 4 is shown the temporal evolution of the eccentricity and of the orbital radius for a satellite with an initial altitude of 500km and using a thruster of 0.2N.

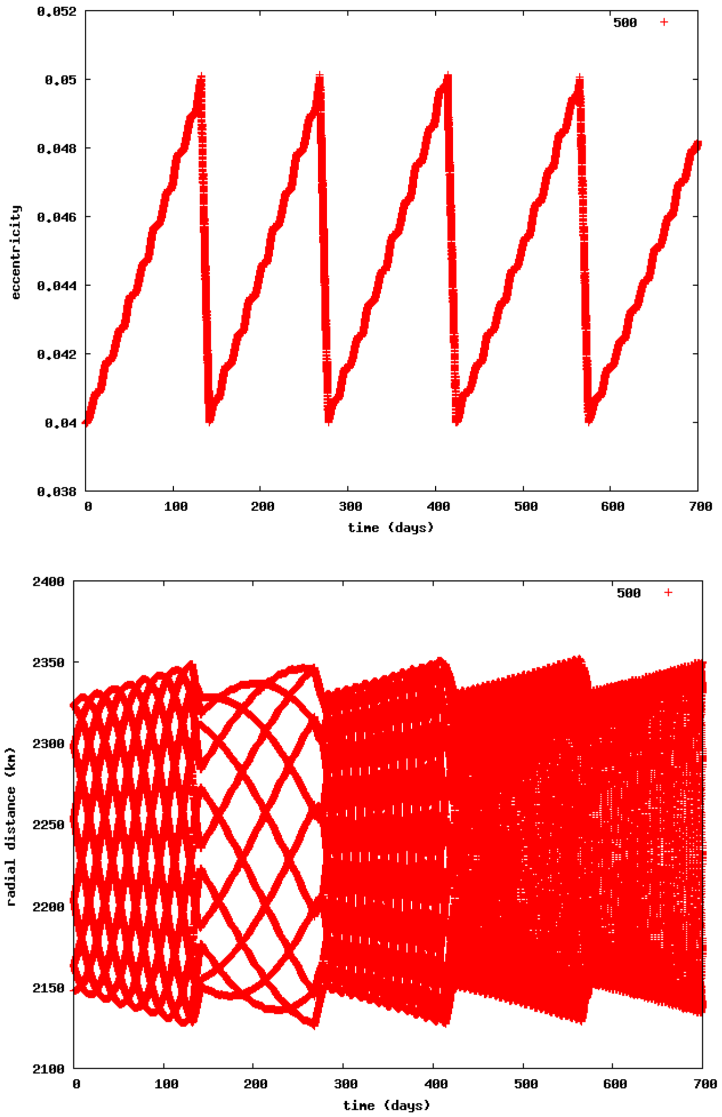

Figure 4. Temporal evolution of the eccentricity (top) and of the orbital radius (bottom) . In this simulation the initial altitude was 500km and the thrust value used was 0.2N.

In Figure 5 we present the fuel consumption per year of lifetime for the whole set of simulations, i.e., different initial altitudes and different values of the thruster. The time interval that the thrusters are turned on and off are shown in Figures 6 and 7.

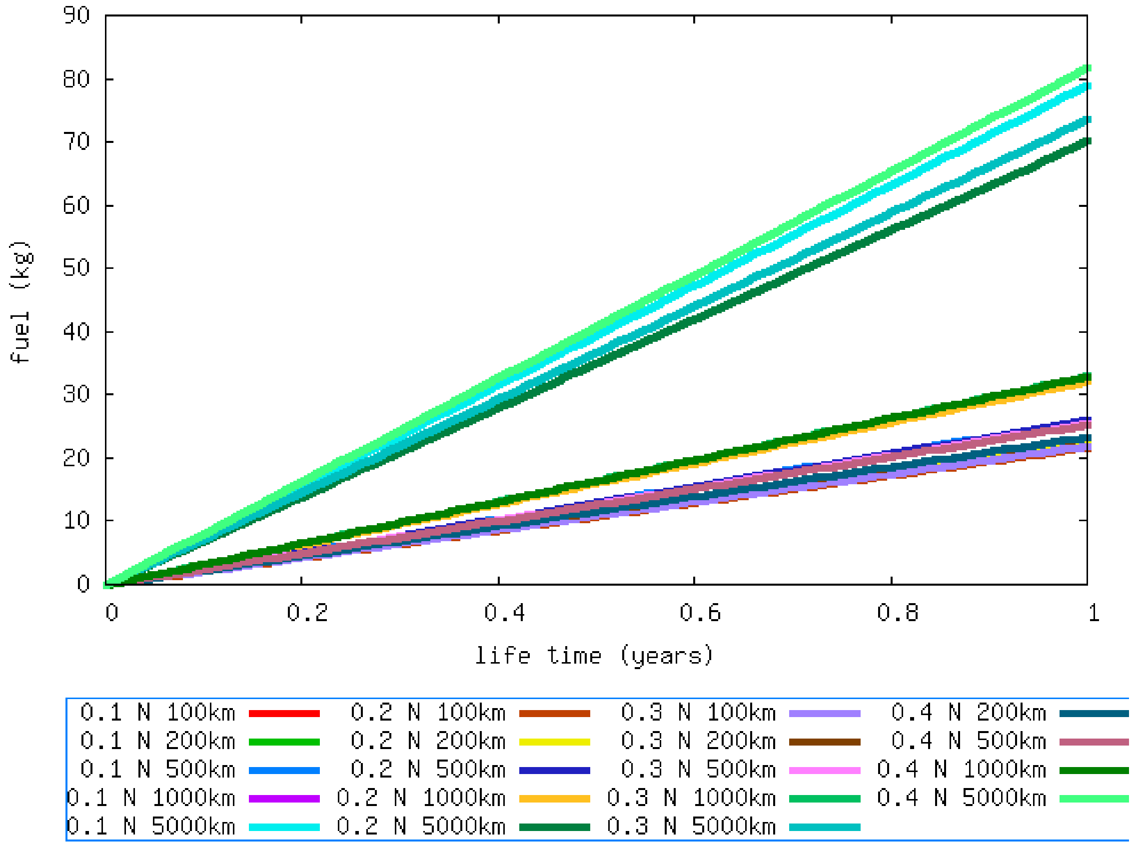

Figure 5. The fuel consumption per year of lifetime for the whole set of simulations, i.e., different initial altitudes and different values of the thruster.

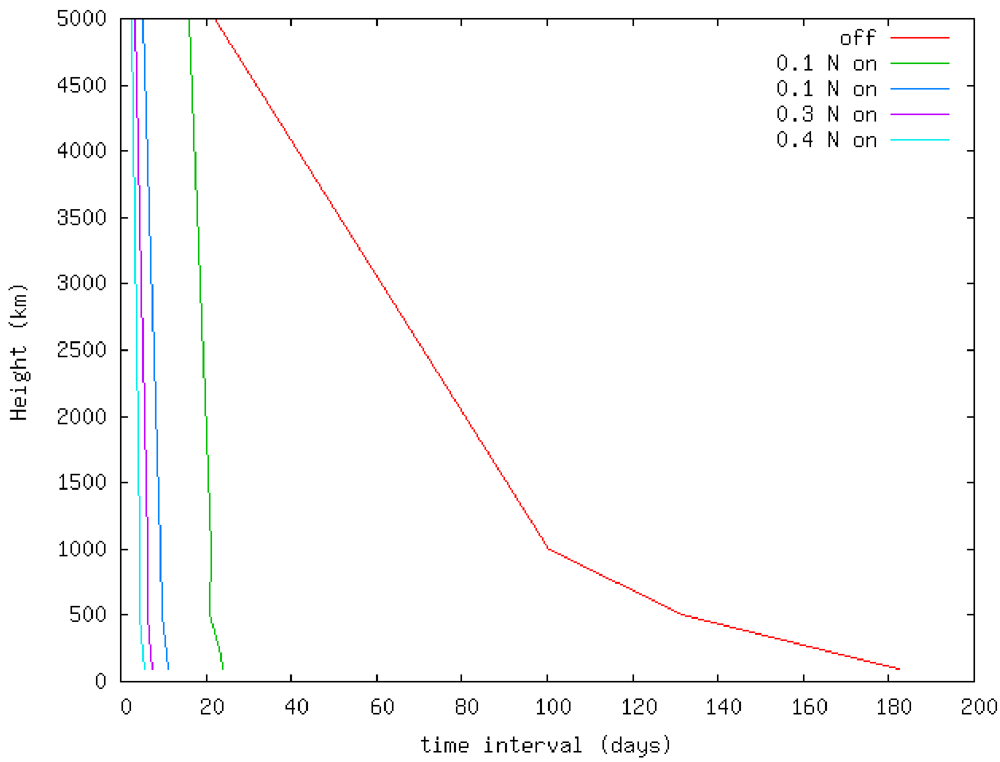

Figure 6. Time interval that the thrusters are turned on and off, for the whole set of simulations.

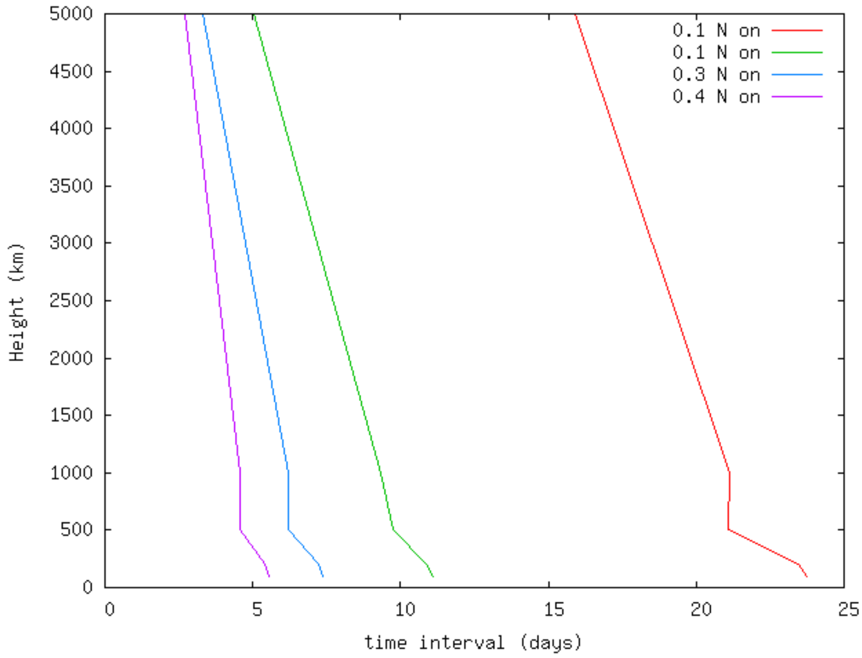

Figure 7. Zoom of Figure 6

5. FINAL COMMENTS

In the present work we have studied the problem of polar lunar satellites in near circular orbits under the gravitational perturbations of the Earth and the Sun. The problem is dominated by the Lidov-Kozai resonance, which forces the satellite's eccentricity to growth exponentially. In order to keep the satellite with low eccentricity we propose to use low thrust propulsion. The results show that the satellite's lifetime can be reasonably extended (several years) at a not so expensive cost. This is an ongoing work.

6. ACKNOWLEDGEMENTS

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